2601/103
2602/103
2603/103
ENGINEERING MATHEMATICS I
June/ July 2016
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN ELECTRICAL AND ELECTRONIC ENGINEERING
(POWER OPTION)
(TELECOMMUNICATION OPTION)
(INSTRUMENTATION OPTION)
MODULE I

ENGINEERING MATHEMATICS I

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:
- *Answer booklet;*
- *Drawing instruments;*
- *Mathematical tables/ non-programmable scientific calculator.*

*This paper consists of **EIGHT** questions.*
*Answer any **FIVE** questions.*
All questions carry equal marks.
Maximum marks for each part of a question are as shown.
Candidates should answer the questions in English.

This paper consists of 4 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

 Turn over

1. (a) Solve the equations:

(i) $5^{6x+1} \times 25^{x-7} = 125$; (4 marks)

(ii) $\log_3 (x+2)^2 = 2$ (4 marks)

(b) Convert:

(i) $r = 4(1 + 2\sin 2\theta)$ to cartesian form.

(ii) $xy = 3$ to polar form.

(6 marks)

(c) Three currents I_1, I_2 and I_3 in amperes flowing in an electric circuit satisfy the following simultaneous equations:

$$3I_1 + 2I_2 + 5I_3 = 2$$
$$3I_1 + 3I_2 - 2I_3 = 4$$
$$2I_1 - 5I_2 - 3I_3 = 14$$

Use elimination method to determine the values of the three currents. (6 marks)

2. (a) Prove the identity $\dfrac{\tan\theta + \sec\theta}{\sec\theta\left(1 + \frac{\tan\theta}{\sec\theta}\right)} = 1$. (4 marks)

(b) If $\sin A = \dfrac{3}{5}$ and $\cos B = \dfrac{6}{10}$, where A and B are acute angles, determine:

(i) $\sin (A - B)$;
(ii) $\cot 2A$.

(6 marks)

(c) Given that $8 \cos \theta + 36 \sin \theta = R \sin (\theta + \alpha)$, where $R > 0$ and $0^\circ \leq \alpha \leq 90^\circ$:

(i) find the values of R and α;

(ii) hence, solve the equation $8\cos\theta + 6\sin\theta = 6$ for $0^\circ \leq \theta \leq 360^\circ$.

(10 marks)

3. (a) A committee of 5 is to be chosen from 7 men and 6 women. Find the number of ways in which the committee can be formed so that it contains at least 3 men.

(5 marks)

(b) (i) Prove that if x^3 and higher power can be neglected, $\sqrt{\frac{1+3x}{1-3x}} = 1 + 3x + \frac{9}{2}x^2$.

(ii) Hence, by letting $x = \frac{1}{9}$ in (i) above, show that $\sqrt{2} = 1\frac{11}{25}$.

(9 marks)

(c) The resonant frequency of a series electric circuit is given by $f_r = \frac{1}{2\pi\sqrt{LC}}$, where L is the inductance and C is the capacitance. If L increases by 2.4% and C decreases by 0.7%, determine using the binomial theorem the percentage change in resonant frequency f_r, correct to one decimal place.

(6 marks)

4. (a) Find the inverse function of $f(x) = \frac{-2}{x-5}$. (4 marks)

(b) (i) Show that $\tanh^{-1}x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$;

(ii) Hence, determine $\tanh^{-1}0.71$, correct to four decimal places.

(8 marks)

(c) Solve the equation $2\sinh x + 3\cosh x = 5$, correct to four decimal places. (8 marks)

5. (a) Given that $Z_1 = 1 + j2$, $Z_2 = 2 - j3$ and $Z_3 = -4 + j12$, determine:

(i) $3Z_1 + Z_2 - Z_3$;

(ii) $Z_1 + \frac{Z_2 Z_3}{Z_2 + Z_3}$.

(7 marks)

(b) Use Demoivre's theorem to express $\cos^6\theta$ in terms of the cosines of multiples of θ.

(6 marks)

(c) (i) If $Z = x + jy$, show that the locus defined by $\arg\left\{\frac{Z+2}{Z}\right\} = \frac{\pi}{4}$ is a circle.

(ii) Hence, determine its centre and radius.

(7 marks)

6. (a) Find $\frac{dy}{dx}$ given that:

(i) $y = x^3 \cos^3 2x$

(ii) $xy^3 + y^3x^3 + 4 = 0$

(iii) $x = 4\sec\theta, y = 3\tan\theta$

(8 marks)

(b) If $y = 8x^2e^{-x}$, show that $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 16e^{-x}$. (4 marks)

(c) Given that $f(x) = 2x^3 - \frac{15}{2}x^2 + 9x + 3$, find the:

(i) coordinates of the turning point;

(ii) hence, determine their nature.

(8 marks)

7. (a) Given that $Z = 2\cos(4x + 5y)$, show that $4\frac{d^2z}{dy^2} - 5\frac{d^2z}{dx^2} = -10z$. (5 marks)

(b) The time of oscillation t of a pendulum is given by $t = 2\pi\sqrt{\frac{L}{g}}$. Use partial differentiation to determine the percentage change in t, if L is increasing at 0.3% and g is decreasing at 0.2%. (6 marks)

(c) Locate the stationary point of the function $f(x,y) = 2x + 2y - 2xy - 2x^2 - y^2 + 4$ and determine their nature. (9 marks)

8. (a) Evaluate the integrals:

(i) $\int \frac{4x^2 - 7x + 13}{(x-2)(x^2+1)}dx$;

(ii) $\int x^4 \ln 2x\, dx$;

(iii) $\int_0^1 \frac{1}{\sqrt{3 - 2x - x^2}}dx$.

(12 marks)

(b) Find the area bounded by the curve $y = 2x^2 + 3x - 4$, the x-axis and the ordinates at $x = 2$ and $x = 4$. (3 marks)

(c) Determine the root mean square value of the function $y = 200 \sin 250\pi t$, between the ordinates $t = 0$ and $t = \frac{1}{100}$, correct to two decimal places. (5 marks)

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