1. (a) Simplify the expressions:

(i) $$\frac{(1-x)^{\frac{1}{2}}-(1-x)^{-\frac{1}{2}}}{(1+x)^{\frac{1}{2}}}$$

(ii) $$\frac{\log 125 - \log 25 + \log 5}{\log 625 + \frac{1}{2}\log 25}$$

(7 marks)

(b) Solve the equation:

$$4^x + 1 = 3 + 2^x$$

(5 marks)

(c) The application of Kirchoff's laws to a d.c. circuit yielded the simultaneous equations:

$$I_1 - 2I_2 + I_3 = 0$$

$$-2I_1 + 3I_2 + 2I_3 = 2$$

$$3I_1 + 4I_2 - 3I_3 = 14$$

Where I_1, I_2 and I_3 are currents in amperes. Use elimination method to solve the equations. (8 marks)

2. (a) Find the coefficient of x^6 in the binomial expansion of $(3x+2y)^{10}$, and determine its value when $x=\frac{1}{2}$ and $y=\frac{1}{3}$. (5 marks)

(b) (i) Determine the first four terms in the binomial expansion of $(3+4x)^{-\frac{1}{2}}$, and state the values of x for which the expansion is valid.

(ii) Use the binomial theorem to expand $\left(1+\frac{1}{4}x\right)^{\frac{1}{3}}$ as far as the term in x^3. Hence determine the value of $\sqrt[3]{65}$, correct to four decimal places. (9 marks)

(c) Solve the equation:

$$3^{2x+1} - 7(3^x) + 2 = 0$$

(6 marks)

3. (a) Given the complex numbers $z_1 = 2+3j$, $z_2 = 1+2j$ and $z_3 = 3-4j$, express

$z_1 + \frac{z_2 z_3}{z_2 + z_3}$ in polar form.

(8 marks)

(b) One root of the equation $z^3 + 4z^2 + kz + 8 = 0$ is $-1 + j\sqrt{3}$. Determine the:

(i) value of k;
(ii) other roots.

(6 marks)

(c) Solve the equation:

$$z^3 - 1 + j\sqrt{3} = 0$$

(6 marks)

4. (a) Given $y = \frac{x}{1-x}$, find $\frac{dy}{dx}$ from first principles. (5 marks)

(b) Use implicit differentiation to determine the equations of the:

(i) tangent;
(ii) normal

to the curve $x^3 + y^2 + 3xy - 2x + 6y + 9 = 0$ at the point $(1, -1)$. (10 marks)

(c) Determine the stationary points of the curve $f(x) = x^3 + 15x^2 + 27x + 2$, and state their nature. (5 marks)

5. (a) (i) Evaluate the indefinite integral

$$\int \frac{\sin 2x}{1 + \sin^2 x}\,dx$$

(ii) Show that $\int_0^1 \frac{x}{(x+1)(x^2+x+1)}\,dx = \ln\left(\frac{\sqrt{3}}{2}\right) + \frac{\pi}{6\sqrt{3}}$

(12 marks)

(b) Use the integration to determine the length of the curve $y = \frac{1}{3}x^{\frac{3}{2}}$ between the points $x = 0$ and $x = 4$. (8 marks)

6. (a) Prove the trigonometric identities:

(i) $\frac{1 - \sin\theta}{\cos\theta} + \frac{\cos\theta}{1 - \sin\theta} = 2\sec\theta$

(ii) $\frac{1 - \cos\theta}{1 + \cos\theta} = (\operatorname{cosec}\theta - \cot\theta)^2$

(9 marks)

(b) Solve the equation:

$3\sin^2\theta + 5\cos\theta = 5$, for values of θ between 0° and 360° inclusive.

(5 marks)

(c) (i) Express $4\cos\theta + 3\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0^\circ \leq \alpha \leq 90^\circ$.

(ii) Hence solve the equation:

$4\cos\theta + 3\sin\theta = 5$ for values of θ between 0° and 180° inclusive.

(6 marks)

7. (a) Given $u = \dfrac{x - 3y}{x + 3y}$, show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = 0$

(5 marks)

(b) The radius of a right circular cone is increasing at a rate of 18 cm/s while its height is decreasing at a rate of 25 cm/s. Determine the rate of change of the volume of the cone when the radius is 120 cm and the height is 140 cm. (4 marks)

(c) Locate the stationary points of the function $z = x^3y + 12x^2 - 8y + 2$, and determine their nature. (11 marks)

8. (a) Prove the identities:

(i) $\sinh(x + y) = \sinh x \cosh y + \cosh x \sinh y$;

(ii) $\tanh 2x = \dfrac{2\tanh x}{1 + \tanh^2 x}$

(7 marks)

(b) (i) Express $\tanh^{-1} x$ in logarithmic form.

(ii) Hence determine the value of $\tanh^{-1}\left(\frac{1}{2}\right)$, correct to four decimal places.

(8 marks)

(c) Solve the equation $3\cosh^2 x - 7\sinh x - 1 = 0$. (5 marks)

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