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**ENGINEERING MATHEMATICS I**

**June/July 2018**

**Time: 3 hours**



**THE KENYA NATIONAL EXAMINATIONS COUNCIL**

**DIPLOMA IN MECHANICAL ENGINEERING  
(PRODUCTION OPTION)  
(PLANT OPTION)**

**DIPLOMA IN AUTOMOTIVE ENGINEERING**

**DIPLOMA IN WELDING AND FABRICATION**

**DIPLOMA IN CONSTRUCTION PLANT ENGINEERING**

**MODULE I**

**ENGINEERING MATHEMATICS I**

**3 hours**

**INSTRUCTIONS TO CANDIDATES**

*You should have the following for this examination:*

*Answer booklet;*

*Mathematical tables/ Non-programmable scientific calculator;*

*This paper consists of EIGHT questions.*

*Answer any FIVE questions in the answer booklet provided.*

*All questions carry equal marks.*

*Maximum marks for each part of a question are as indicated.*

*Candidates should answer the questions in English.*

**This paper consists of 4 printed pages.**

**Candidates should check the question paper to ascertain that  
all the pages are printed as indicated and that no questions are missing.**

1. (a) Simplify the expressions:

(i) 
$$\frac{(1+x)^{\frac{1}{2}} - (1+x)^{\frac{1}{3}}}{(1+x)^{\frac{1}{6}}}$$

(ii) 
$$\frac{\log 625 - \frac{1}{2} \log 25}{\log 125 + \frac{1}{2} \log 25}$$

(7 marks)

- (b) Solve the equations:

(i)  $2^{2x+1} = 8^{\frac{x}{2}}$

(ii)  $\log_2 4 + 2 \log_2 x^2 = 6$

(13 marks)

2. (a) Find the ratio of the term in  $x^5$  to the term in  $x^7$  in the binomial expansion of  $(2x+5)^{10}$ , and determine its value when  $x = \frac{1}{3}$ , correct to four decimal places.

(8 marks)

- (b) Determine the first four terms in the binomial expansion of  $(1+2x)^{\frac{1}{2}}$ , and state the values of  $x$  for which the expansion is valid.

(4 marks)

- (c) (i) Use the binomial theorem to show that, if  $x$  is very small, then

$$\sqrt{\frac{1 - \frac{1}{2}x}{1 + \frac{1}{2}x}} = 1 - \frac{1}{2}x + \frac{1}{8}x^2 - \frac{1}{16}x^3$$

- (ii) By setting  $x = \frac{1}{2}$  in the result in (i), determine the appropriate value of  $\sqrt{0.6}$ , correct to four decimal places.

(8 marks)

3. (a) Solve the equations:

(i)  $\frac{3}{x-2} + \frac{2}{x-3} = 2$

(ii)  $3(2^{2x}) - 7(2^x) + 2 = 0$

(12 marks)

- (b) Three forces  $F_1$ ,  $F_2$  and  $F_3$  in newtons necessary to keep a certain mechanical system in equilibrium satisfy the simultaneous equation:

$$2F_1 - F_2 + F_3 = 3$$

$$-F_1 + 2F_2 + 2F_3 = -3$$

$$3F_1 - 2F_2 + F_3 = 2$$

Use the method of elimination to determine the values of the forces.

(8 marks)

4. (a) Prove the identities:

(i) 
$$\frac{1 + \cos \theta}{\sin \theta} = \frac{\sin \theta}{1 - \cos \theta}$$

(ii) 
$$\cos \theta + \cos 2\theta + \cos 3\theta = \cos 2\theta (2 \cos \theta + 1)$$

(6 marks)

- (b) (i) Given  $\sin(\theta + \alpha) = 2 \cos(\theta - \alpha)$ , show that  $\tan \theta = \frac{2 + \tan \alpha}{1 + 2 \tan \alpha}$ .

- (ii) Hence solve the equation

$$\sin\left(\theta + \frac{\pi}{4}\right) = 2 \cos\left(\theta - \frac{\pi}{4}\right), \text{ for values of } \theta \text{ between } 0^\circ \text{ and } 360^\circ \text{ inclusive.}$$

(6 marks)

- (c) Solve the equation:

$$3 \cos 2\theta + \sin \theta + 2 = 0, \text{ for values of } \theta \text{ between } 0^\circ \text{ and } 360^\circ \text{ inclusive.}$$

(8 marks)

5. (a) Determine the values of M and N such that  $5 \cosh x - 3 \sinh x = Me^x + Ne^{-x}$ .

(4 marks)

- (b) (i) Derive the identity:

$$\cosh(x + y) = \cosh x \cosh y + \sinh x \sinh y.$$

- (ii) Use Osborne's rule to derive the identity for  $\coth^2 x$  from the trigonometric identity:  $1 + \cot^2 x = \operatorname{cosec}^2 x$ .

(8 marks)

- (c) Solve the equation:

$$3 \cosh 2x - \sinh x - 7 = 0$$

(8 marks)

Handwritten work for question 5(c):

$$3(\cosh^2 x + \sinh^2 x) - \sinh x - 7 = 0$$

$$3(1 + \sinh^2 x) + 3\sinh^2 x - \sinh x - 7 = 0$$

$$3(1 + \sinh^2 x) + 3\sinh^2 x - \sinh x - 7 = 0$$

$$3 - 3\sinh^2 x + 3\sinh^2 x - \sinh x - 7 = 0$$

$$3 - \sinh x - 7 = 0$$

$$\sinh x = -4$$

$$\frac{e^x - e^{-x}}{2} = -4$$

$$e^x - 1 = 8e^{-x}$$

Handwritten work for question 5(b)(ii):

$$\frac{e^x - 8e^{-x} - 1}{2} = 0$$

$$e^x - 8e^{-x} - 1 = 0$$

$$e^{2x} - 8 - e^x = 0$$

$$e^{2x} - e^x - 8 = 0$$

$$\frac{e^x \pm \sqrt{1 + 32}}{2} = 0$$

$$\frac{e^x \pm 5.7446}{2} = 0$$

$$e^x = -2.8723$$

$$e^x = 8.1223$$

6. (a) Given the function  $f(x) = \frac{3-x}{x+4}$ , determine:

(i)  $f^{-1}(0)$

(ii)  $f^{-1}\left(-\frac{1}{2}\right)$

(7 marks)

(b) (i) Show that  $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$ .

(ii) Hence determine the value of  $\tan^{-1}(1) + \tan^{-1}(\sqrt{3})$ .

(8 marks)

(c) By expressing  $\sinh^{-1}x$  in logarithmic form, determine the value of  $\sinh^{-1}(6)$ .

(5 marks)

7. (a) Given the complex numbers  $Z_1 = -1 + 2j$ ,  $Z_2 = 1 + j$  and  $Z_3 = \frac{1}{Z_1} + \frac{1}{Z_2}$ , express  $Z_3$  in polar form.

(8 marks)

(b) Given that  $Z = j$  is one root of the equation  $Z^3 + 3Z^2 + Z + 3 = 0$ , determine the other roots.

(5 marks)

(c) Solve the equation:

$$Z^4 + 1 + j\sqrt{3} = 0, \text{ giving the answers in polar form.}$$

(7 marks)

8. (a) The sum of the first three terms of an arithmetic progression is 3, and the difference between the seventh term and the fourth term is  $-6$ . Determine the:

(i) first term and common difference

(ii) sum of the first thirty terms.

(6 marks)

(b) The third term of a geometric progression is eight times the sixth term, and the sum of the second and fifth terms is  $\frac{9}{16}$ . Determine the:

(i) first term and common ratio

(ii) sum of the first ten terms.

(7 marks)

(c) Express the equation of the parabola  $y^2 = 4 - 4x$  in polar form.

(7 marks)

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