2521/102 2602/103
2601/103 2603/103
ENGINEERING MATHEMATICS I
June/July 2020
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN ELECTRICAL AND ELECTRONIC ENGINEERING
(POWER OPTION)
(TELECOMMUNICATION OPTION)
(INSTRUMENTATION OPTION)

MODULE I

ENGINEERING MATHEMATICS I

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:
- *Answer booklet;*
- *Drawing instruments;*
- *Mathematical tables/Non-programmable scientific calculator.*

*This paper consists **EIGHT** questions.*
*Answer any **FIVE** questions.*
All questions carry equal marks.
Maximum marks for each part of a question are as indicated.
Candidates should answer the questions in English.

This paper consists of 4 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

1. (a) Express the equation of the parabola $y^2 = 8 - 4x$ in polar form. (5 marks)

(b) Solve the equation $4^{2x} - 4^{x+1} + 3 = 0$, correct to three decimal places. (7 marks)

(c) Three currents I_1, I_2 and I_3 in amperes flowing in an electric circuit satisfy the simultaneous equations:

$2I_1 + 3I_2 - 4I_3 = -4$
$3I + 4I_2 - I_3 = 8$

$$2I_1 + 3I_2 - 4I_3 = -4$$
$$3I_1 + 4I_2 - I_3 = 8$$
$$I_1 - 5I_2 + I_3 = -6$$

Use the method of elimination to determine the values of the currents. (8 marks)

2. (a) Simplify the expressions:

(i) $$\frac{(1-x)^{\frac{1}{2}} - (1-x)^{-\frac{1}{2}}}{(1-x)^{\frac{1}{2}}}$$

(ii) $$\frac{\log 125 - \frac{1}{2}\log 25 + \log 625}{\log 3125 + \frac{1}{2}\log 25}$$ (7 marks)

(b) Solve the equations:

(i) $13.2(12^{2x+1}) = 16$, correct to four decimal places.

(ii) $\log_x 2 - \log_4 x + \frac{7}{6} = 0$. (13 marks)

3. (a) Given the functions $f(x) = 9x$ and $g(x) = x + 2$, determine:

(i) $fg(x)$

(ii) $(fg)^{-1}(9)$. (6 marks)

(b) By expressing $\sinh^{-1}x$ in logarithmic form, determine the value of $\sinh^{-1}(0.2)$. (7 marks)

(c) Solve the equation $4\cosh 2x - \sinh 2x = 4$ correct to three decimals places. (7 marks)

4. (a) Five components are to be chosen from 7 resistors and 6 diodes. Determine the number of ways in which the components can be selected so that there are at least 3 resistors in the choice. (5 marks)

(b) Find the term in x^4 in the binomial expansion of $(3x-2)^{11}$, and determine its value when $x = \frac{1}{10}$, correct to three decimal places. (5 marks)

(c) (i) Use the binomial theorem to expand $\left(\frac{1-x}{1+2x}\right)^{\frac{1}{3}}$ up to the term in x^3.

(ii) Hence, evaluate $\left(\frac{0.8}{1.4}\right)^{\frac{1}{3}}$, correct to three decimal places. (10 marks)

5. (a) Differentiate $f(x) = \frac{1}{4x}$, from first principles. (5 marks)

(b) Given that $z = \frac{x+y}{x-y}$, show that $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \frac{4(x+y)}{(x-y)^3}$. (6 marks)

(c) Locate the stationary points of the function $z = x^3 - 9x^2 - 4y^2$ and determine their nature. (9 marks)

6. (a) Given that $\sin A = \frac{4}{5}$ and $\cos B = \frac{8}{10}$, where A and B are acute angles, determine the values of:

$\sin A = 4/5$ $\sin B = 6/10$ $\cos A = 3/5$ $\cos B = 8/10$

(i) $\cos(A-B)$ $\Rightarrow \cos A \cos B - \sin A \sin B$

$\left(\frac{3}{5}\times\frac{8}{10}\right) - \left(\frac{4}{5}\times\frac{6}{10}\right) = \frac{24}{75} = 0.85$

(ii) $\cot 2B$ (6 marks)

$\cot 2B$
$\cot = 1/\tan$
$\tan = \frac{\sin}{\cos}$
$= \frac{\sin 2B}{\cos 2B}$
$= \frac{\sin B + \sin B}{\cos B + \cos B}$
Sijamaliza

(b) Solve the equation $3\cos 2\theta - \sin\theta + 2 = 0$, for values of θ between $0°$ to $360°$ inclusive. (7 marks)

(c) The angle of depression of a ship viewed from the top of a 65 metre vertical cliff is $22°$. If the ship sails away from the cliff a distance x metres, the angle of depression from the top of the cliff is $17°$. Determine the distance x. (7 marks)

7. (a) Evaluate the intergrals:

4(c) $\frac{(1-x)^{\frac{1}{3}}}{1+x}$

$\frac{1-x}{1+x} \times \frac{1-2x}{1-2x}$

$\frac{(1-x)(1-2x)}{1(1+2x)+2x(1-2x)}$

$\frac{1-2x-x+x^2}{1-2x+2x-4x^2}$

$\frac{1-3x+2x^2}{1-4x^2} = 0$

$1-3x+2x^2 = 1-4x-2$
$= 3x$

(i) $\int_0^{\frac{\pi}{2}} \frac{3}{1+\cos x}\,dx$

(ii) $\int_0^{\pi} x^2 \sin x\,dx$

(iii) $\int_1^2 \frac{2}{x^2\sqrt{1+x^2}}\,dx$ (12 marks)

(b) Sketch the region bounded by the curve $y = x^2 - 2$ and the line $y = -4 - 3x$ and use integration to determine its value. (8 marks)

6(b) $3\cos 2\theta - \sin\theta + 2 = 2$
$\cos 2\theta = 1 - 2\sin^2\theta$
$3(1-2\sin^2\theta) = 2$
$3 - 6\sin^2\theta = 2$

Achia hapo ataka wanitafuta

8. (a) Given the complex numbers $z_1 = 2 + j$ and $z_2 = 5j$, determine $\frac{z_2}{z_1}$, expressing the a answer in exponential form. (6 marks)

(b) Use DeMoivre's theorem to show that $\sin^5\theta = \frac{1}{16}(\sin 5\theta - 5\sin 3\theta + 10\sin\theta)$. (6 marks)

(c) If $z = x + jy$, determine the equation of the locus defined by $\arg\left(\frac{z+3}{z-2}\right) = \frac{\pi}{4}$. (8 marks)

6(b) $3(\cos^2\theta - \sin^2\theta) - \sin\theta + 2 = 0$

$3\cos^2\theta - 3\sin^2\theta - \sin\theta + 2 = 0$

$3 - 3\sin^2\theta - 3\sin\theta - \sin\theta + 2 = 0$

$3 - 6\sin^2\theta - \sin\theta + 2 = 0$

$6\sin^2\theta + \sin\theta - 5 = 0$

let $\sin\theta$ = the solve equation.

THIS IS THE LAST PRINTED PAGE.

4 (b)

$(3x-2)^{14}$

$-2^{14} + -2^{13}\cdot 3x^1 + -2^{12}\cdot 3x^2 + -2^{11}\cdot 3x^3 + -2^{10}\cdot 3x^4 + -2^{9}\cdot 3x^5 + -2^{8}\cdot 3x^6$

$16384 + -8192(3x) + 36864x^2 + -55296x^3 + 82944x^4 - 124416x^5 + 186624x^5$

$= 16384 - 24756x + 36864x^2 - 55296x^3 + 82944x^4 + 124416x^5 + 186624x^5$

$1(16384) - 14(24756x) + 91(36864x^2) - 364(55296x)^3 + 1001(82944x^4) - 2002(124$

$+ 3003(186624x)^6$

$= 16384 - 344064x + 3354624x^2 - 20127744x^3 + 8302694$

$249080832x^5 + 56043172x^6$

$x = \frac{1}{10}$

$16384 - 344064\left(\frac{1}{10}\right) + 3354624\left(\frac{1}{10}\right)^2 - 20127744\left(\frac{1}{10}\right)^3$

~~-16384~~

$= 1768.4139 \ldots - 1760 \ldots$