2405/301
MATHEMATICS
Oct./Nov. 2017
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN APPLIED STATISTICS

MATHEMATICS

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:

- *Answer booklet;*
- *Mathematical tables / Scientific calculator.*

*This paper consists of **EIGHT** questions.*
*Answer any **FIVE** questions.*
All questions carry equal marks.
Maximum marks for each part of a question are indicated.
Candidates should answer the questions in English.

This paper consists of 4 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

1. (a) Find the inverse of the matrix

$$A=\begin{bmatrix} 4 & 3 & -3 \\ 5 & 4 & 4 \\ -2 & 2 & -2 \end{bmatrix}$$

Verify that $A^{-1}A = I$, where I is the identity matrix.

(10 marks)

(b) Use Cramer's rule to solve the following simultaneous equations:

$$15x - 20y - 10z = 2$$
$$10x + 25y - 10z = 16$$
$$5x + 10y + 5z = 9$$

(10 marks)

2. (a) If $e^{x+y} = x^2y^2$ find an expression for $\frac{dy}{dx}$, using partial differentiation. (5 marks)

(b) $P = W^2hd$. If errors of upto ±1% are possible in the measured values of W, h and d, find the maximum possible error in the calculated value of P.

(8 marks)

(c) Solve the equation $\frac{\partial^2 U}{\partial x \partial y} = \sin(x+y)$

given that at $y = 0$, $\frac{\partial U}{\partial x} = 1$ and at $x = 0$, $U = (y-1)^2$ (7 marks)

3. (a) Solve the differential equation $(x^3 + y^3) = 6xy^2\frac{dy}{dx}$

given that when $y = 1$, $x = 1$. (9 marks)

(b) In a galvanometer scale, the deflection θ satisfies the differential equation:

$$\frac{d^2\theta}{dt^2} + 8\frac{d\theta}{dt} + 16\theta = 8$$

Solve the equation for θ, given that when t = 0, $\theta = 0$ and $\frac{d\theta}{dt} = 0$. (11 marks)

$f_0 = -0.932 \quad \Delta f_0 = 0.031 \quad \Delta^2 f_0 = -0.03 \quad \Delta^3 f_0 = 0.03 \quad \Delta^4 f_0 = 0 \quad \Delta^5 f_0 = 0.12$

4. (a) Evaluate

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^5 x\,dx$$

(7 marks)

(b) The curve $y = 3x^2 + 5$ is rotated 360° about the x-axis between the ordinates $x = 1$ and $x = 2$.

Determine the:

(i) volume of solid generated;

(ii) the centroid of the solid.

$V = \int_a^b \pi y^2\,dx$

$\bar{x} = \frac{\int_a^b xy\,dx}{\int_a^b y\,dx} \qquad \bar{y} = \frac{\frac{1}{2}\int_a^b y^2\,dx}{\int_a^b y\,dx}$

(13 marks)

5. (a) If $p = 2i + 5j - 2k$ and $q = 4i + j - 6k$ determine

(i) $p \times q$

(ii) the angle between p and q.

(7 marks)

(b) Table 1, gives data obtained in an experiment.

Table 1

x	-0.3	-0.2	-0.1	0.00	0.1	0.2	0.3	0.4	0.5
f(x)	-1.143	-0.932	-0.901	-0.900	-0.899	-0.868	-0.657	0.124	2.225

Use Gregory-Newton formula to evaluate, correct to four significant figures

(i) f(-0.18)

(ii) f(0.42)

(13 marks)

6. (a) Evaluate

(i) $\frac{20\angle 30^\circ \times 10\angle 45^\circ}{5\angle 60^\circ}$

(ii) $5\angle 30^\circ + 4\angle -60^\circ - 6\angle -135^\circ$

giving answers in polar and in cartesian forms, correct to three significant figures.

(11 marks)

(b) Determine the **four** roots of $(-5-j7)^{\frac{1}{4}}$ in polar forms. (9 marks)

7. (a) Find the middle term of the expansion $(2x + 3)^8$, hence the value of this term when $x = \frac{1}{12}$. (8 marks)

(b) (i) Find the first **three** non-zero terms in the Maclaurin's expansion of $\sin\theta$.

(ii) Hence evaluate $\int_0^{\frac{\pi}{4}} \frac{\sin\theta}{\theta} d\theta$ giving the answer correct to 3 significant figures.

(12 marks)

8. (a) Use the mid-ordinate rule with six intervals to evaluate

$$\int_0^{\frac{\pi}{4}} \frac{1}{1+\sin\theta} d\theta$$

(10 marks)

(b) Evaluate

$$\int_0^{\frac{\pi}{4}} \tan x \, dx$$

by Simpson's rule with **four** strips.

(10 marks)

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