2405/301
MATHEMATICS
June/July 2019
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN APPLIED STATISTICS

MATHEMATICS

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:
Answer booklet;
Mathematical tables/Scientific calculator.
*This paper consists of **EIGHT** questions.*
*Answer any **FIVE** questions.*
***ALL** questions carry equal marks.*
Maximum marks for each part of a question are indicated.
Candidates should answer the questions in English.

This paper consists of 4 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

1. (a) Given that matrices: $A=\begin{pmatrix}3 & -1 & 2\\ 1 & -4 & 1\\ 2 & 3 & 3\end{pmatrix}$, $B=\begin{pmatrix}7 & 2\\ -1 & 3\\ 4 & 1\end{pmatrix}$ and $C=\begin{pmatrix}1 & 2 & 4\\ 3 & 6 & 2\end{pmatrix}$;

evaluate:

(i) $BC+A$;

(ii) $(CA)B$. (7 marks)

(b) Use the inverse matrix method to solve the simultaneous equations:

$$6k+5j+4i=1$$
$$i+2j+3k=-2$$
$$j+2k+4i=5$$

(13 marks)

2. (a) Solve the differential equation:

$$(x-y)\frac{dy}{dx}=x+y$$

(7 marks)

(b) Solve the following differential equation using the D-operator method:

$$\frac{d^2y}{dx^2}+3\frac{dy}{dx}+2y=\sin 2x$$

Given that at $x=0$, $y=0$ and $\frac{dy}{dx}=0$. (13 marks)

3. (a) Given that $z_1=4\angle 30°$

$z_2=3\angle -60°$

$z_3=5\angle -135°$

54∠10° 12.3 13

Determine the following giving the answer in polar form:

(i) $\frac{z_1 z_2}{z_3}$;

(ii) $z_1+z_2-z_3$. (7 marks)

(b) Use Demoivre's theorem to show that $\cos^4\theta=\frac{1}{8}\cos 4\theta+\frac{1}{2}\cos 2\theta+\frac{3}{8}$. (6 marks)

(c) Determine the fourth root of $z=\sqrt{4-3j}$. (7 marks)

4. (a) (i) Use Maclaurin's series expansion to determine the first four terms of the series for $f(x)=\cos 2x$.

(ii) Hence deduce the series for $\cos^2 x$. (12 marks)

$f(0)+f'(0)x+\frac{f''(0)x^2}{2!}+\frac{f'''(0)x^3}{3!}+\frac{f''''(0)x^4}{4!}$

$\cos^2\theta=1-\sin^2\theta$

$3.00+3.9999j$

$-3.9999+3.000j=(4-3j)^{1/4}$

(b) (i) Use Taylor's Theorem to expand $f(x)=3x^3-6x^2+4x+6$ in ascending powers of $(x-1)$ up to and including $(x-1)^3$.

(ii) Hence evaluate:

$$\int_2^3 \frac{3x^3-6x^2+4x+6}{(x-1)^3}\,dx$$, giving the answer correct to 4 decimal places.

(8 marks)

5. (a) Derive the laws of logarithm in the following forms:

(i) $\log PQ=\log P+\log Q$;

(ii) $\log P/Q=\log P-\log Q$;

(iii) $\log P^n=n\log P$.

(9 marks)

(b) Execute the following binary operations then convert the answers to denary numbers:

(i) 111011 + 101111;

(ii) 10111 x 1101.

(6 marks)

(c) Solve for x in the equation:

$3\cos 2x+5\sin x=4$ if $90^\circ\le x\le 180^\circ$.

(5 marks)

6. (a) Given that $z=\tan^{-1}\left(\frac{2xy}{x^2-y^2}\right)$, show that $\frac{\partial^2 z}{\partial^2 x}+\frac{\partial^2 z}{\partial^2 y}=0$. (8 marks)

(b) Given the function $z=x^3-6x^2-8y^2$, determine the stationary values of z and hence classify them.

(12 marks)

7. (a) Evaluate:

(i) $\int_0^2\int_0^2 e^{x^2}\,dy\,dx$

(ii) $\int_1^2\int_0^3 \frac{y}{1+y^2}\,dx\,dy$

(10 marks)

$x^2(x-4)$ $2(x^2-4y^2)$

$2y+2x=0$

$2y=2x$

(b) Use double integrals to determine the area bounded by the graphs $y=2+x-x^2$ and $y=x-2$.

(10 marks)

$y=x$

$y-x=0$

$y=x$

$f(a)+f'(a)h+\frac{f''(a)h^2}{2!}+\frac{f'''(a)h^3}{3!}+\frac{f^{iv}(a)h^4}{4!}$

$(x-1)(x-1)$

$x^2-x-x+1$

$(x^2-2x+1)(x-1)$

$3x$

8. (a) Using Newton-Raphson method, solve the equation $x^3 + x - 1 = 0$, near $x = 1$, correct to three decimal places. (9 marks)

(b) Table 1 shows values of x and $f(x)$ for a given function.

Table 1

x	-3	-2	-1	0	1	2	3
$f(x)$	-83	-30	-1	10	9	2	-5

Apply the Newton-Gregory method to evaluate:

$X_p = X_1 - \frac{f(x}{f(x}$

(i) $f(-1.5)$;

(ii) (2.5). (11 marks)

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