1503/203
MATHEMATICS II
Oct./Nov. 2021
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

CRAFT CERTIFICATE IN AUTOMOTIVE ENGINEERING

MODULE II

MATHEMATICS II

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:
Answer booklet;
Scientific calculator;
Drawing instruments.
This paper consists of ***EIGHT*** *questions.*
Answer ***FIVE*** *questions in the answer booklet provided.*
All questions carry equal marks.
Maximum marks for each part of a question are as shown.
Candidates should answer the questions in English.

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This paper consists of 4 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

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1. (a) (i) Use the binomial theorem to expand

$$\sqrt{\frac{1+5x}{1-5x}}$$ as far as the term in x^3.

(ii) By setting $x=\frac{1}{20}$, determine the approximate value of $\sqrt{15}$ correct to 5 decimal places.

(10 marks)

(b) Use elimination method to solve the equations

$$3x+5y+2z=38$$
$$6x-2y+5z=34$$
$$4x+3y+z=26$$

(10 marks)

2. (a) Evaluate the integrals:

(i) $$\int_0^1\left[\frac{x(3x^2+5x+7)}{x^{\frac{1}{4}}}\right]dx$$

(ii) $$\int_0^{\pi}\cos^2 x\,dx$$ correct to 3 decimal places.

(10 marks)

(b) Determine the area bounded by the curve

$y=6x-x^2$, the x - axis and the line $y=3x-4$. (10 marks)

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3. (a) Determine the angle between the vectors

$\underline{A}=2\underline{i}+4\underline{j}+5\underline{k}$ and $\underline{B}=4\underline{i}+6\underline{j}-7\underline{k}$. (6 marks)

(b) Given the scalar field $\phi(x,y,z)=x^4+y^4+z^4$. Determine the directional derivative of ϕ at the point $A(1,-2,1)$ in the direction of AB, where B is $(2,6,-1)$. (7 marks)

(c) A particle's displacement at any time t is given by $x=2t^3-4t$, $y=5t^2+4t$ and $z=8t^2-3t^3$. Determine at $t=2$ the:

(i) velocity;

(ii) acceleration of the particle.

(7 marks)

4. (a) Given the matrix $A = \begin{bmatrix} 6 & -6 & -12 \\ -3 & 9 & 12 \\ 3 & -6 & -9 \end{bmatrix}$, determine $B = A^3 - 2A^2$. (8 marks)

(b) Use the inverse matrix method to solve the equations:

$$\begin{aligned} 4x + 5y - 3z &= 4 \\ 3x + 2y + z &= 14 \\ -5x + y + z &= 3 \end{aligned}$$

(12 marks)

5. (a) Given that $V(x,y) = (x^3 + y^3) - 4Mx^2y^2$ where M is a constant, show that

$$\frac{x^2 \partial^2 V}{\partial x^2} - y^2 \frac{\partial^2 v}{\partial y^2} = 6(x^3 - y^3).$$

(6 marks)

(b) The radius of a cone decreases by 4% while the height increases by 5%. Use partial differentiation to determine the change in volume. (5 marks)

(c) Locate stationary points of the function $f(x, y) = 2x^2 + 7xy + 5y^2 - 69x - 105y + 21$ and determine their nature. (9 marks)

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6. (a) Given that $\cos\theta = \frac{5}{13}$, determine the other five trigonometric ratios of θ. (6 marks)

(b) Prove the identity:

$$\frac{\sin\theta}{1 + \sec\theta} - \frac{\sin\theta}{1 - \sec\theta} = 2\cot\theta.$$

(6 marks)

(c) Solve the equation: $2\sin^2\theta = -3\cos\theta$ for values of θ between 0° and 360° inclusive. (8 marks)

7. (a) Determine the values of S and R such that $R\cosh 2x + S\sinh 2x = 4e^{2x} + 7e^{-2x}$. (5 marks)

(b) Determine the logarithmic form of $\tanh^{-1} x$. (7 marks)

(c) Solve the equation:

$$2\sinh 2x + 3\cosh 2x = 4.$$

(8 marks)

8. (a) Find $\frac{dy}{dx}$ of the function $y = \sin x$ from first principles. (6 marks)

(b) Use implicit differentiation to determine the equation of the normal to the function $x^3 + 2xy + 5y^2 = 8$ at the point (1, 1).

(7 marks)

(c) A particle moves from rest to the point R such that its displacement is given by $S = \frac{t^3}{3} + \frac{7}{2}t^2 + 12t + 8$. Determine:

(i) the displacement at $t = 2$.

(ii) the velocity at $t = 2$.

(iii) when its momentarily at rest.

(7 marks)

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