1. (a) Three co-planar vectors are given by:

$$\underset{\sim}{A} = 4\underset{\sim}{i} + 2\underset{\sim}{j} + 8\underset{\sim}{k}$$

$$\underset{\sim}{B} = x\underset{\sim}{i} + 2\underset{\sim}{j} + 7\underset{\sim}{k}$$

$$\underset{\sim}{C} = \underset{\sim}{i} + 4\underset{\sim}{j} + 2\underset{\sim}{k}$$

Determine the value of x. (6 marks)

(b) Given $\phi = 2xy^2 + y^2z + x^2z - 11 = 0$

and $\underset{\sim}{A} = xyz\underset{\sim}{i} + (xy - 2yz)\underset{\sim}{j} + yz^2\underset{\sim}{k}$

Determine at the point (1, 1, 8)

(i) $\nabla\phi$ (4 marks)

(ii) $\nabla . \underset{\sim}{A}$ (4 marks)

(iii) $\nabla \times \underset{\sim}{A}$

(6 marks)

2. (a) Determine the matrix A:

$$A\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ -1 \end{pmatrix} \; and \begin{pmatrix} 3 \\ -1 \end{pmatrix} = \begin{pmatrix} 11 \\ -11 \end{pmatrix}$$

(9 marks)

(b) Solve by Crammer's rule the equations

$x - y + z = 3$
$3x - 9y + 5z = 6$
$x - 3y + 3z + 13$ (11 marks)

3. (a) Solve the differential equation $4x^2 \dfrac{dy}{dx} = x^2 + 3y^2$. (10 marks)

(b) An electric current, x in a circuit varies with time, t such that $\dfrac{d^2x}{dt^2} + 4\dfrac{dx}{dt} + 3x = 2e^{-2t}$.

Solve for x given that when t = 0, $x = 0$, $\dfrac{dx}{dt} = 0$. (10 marks)

4. (a) Derive from the first principle the Laplace transform of f(t) = Cos t.

(6 marks)

(b) Apply Laplace transforms to solve the differential equation

$\dfrac{d^2x}{dt^2} - 6\dfrac{dx}{dt} + 9x = t$ given $x(0)$ and $\dfrac{dx}{dt} = -1$ when t = 0.

(14 marks)

5. (a) The torque, T produced by a motor is given by:
$T = 1000Bi$ Nm where B is magnetic flux density i is the electric current. At one instance, B was 50 Teslas while i was 10 Amperes. Apply partial derivatives to approximate the change in torque if B rises by 2 Teslas while i drops by 3 Amperes.

(6 marks)

(b) Determine and classify the stationery points of the function.
$f(x,y) = 3y^3 - 2xy + 5y^2 - 4x^2 + 6$ (14 marks)

6. The lengths of metallic conduits in a shop were measured and recorded in a frequency table as shown in table 1.

Length (metres)	29.5 - 29.6	29.7- 29.8	29.9 - 30.0	30.1 - 30.2	30.3 - 30.4	30.5 - 30.6	30.7 - 30.8
Frequency	3	6	11	5	5	3	2

Table 1

From the data, determine:
(a) the mean length
(b) the standard deviation.
(c) the median.
(d) the mode.

(20 marks)

7. (a) A firm purchased 800 cartons of bolt each of which contain 10 bolts. If a carton picked at random has 5% defective bolts, determine:
(i) the expected number of defective bolts in the 800 cartons.
(ii) the number of cartons likely to contain no defective bolts.
(iii) the number of cartons likely to contain at least three defective bolts.

(9 marks)

(b) A continuous random variable x has a probability density function $f(x)$ defined by:

$$f(x) = \begin{cases} k(x^2 - x), & 0 < x < 3 \\ 0, & elsewhere \end{cases}$$

Determine:
(i) the value of constant k.
(ii) the mean and the standard deviation of x. (11 marks)

8. (a) Expand by Maclaurin's series the function $f(x) = \sin 2x$ upto the term in x^5, hence approximate the value of sin 2° to three significant figures. (12 marks)

(b) Apply Taylor's theorem to approximate tan 46° using the first three terms giving your answer correct to four decimal places. (8 marks)

TABLE OF LAPLACE TRANSFORM FORMULAS

$\mathcal{L}[t^n] = \dfrac{n!}{s^{n+1}}$ $\qquad$ $\mathcal{L}^{-1}\left[\dfrac{1}{s^n}\right] = \dfrac{1}{(n-1)!}t^{n-1}$

$\mathcal{L}[e^{at}] = \dfrac{1}{s-a}$ $\qquad$ $\mathcal{L}^{-1}\left[\dfrac{1}{s-a}\right] = e^{at}$

$\mathcal{L}[\sin at] = \dfrac{a}{s^2+a^2}$ $\qquad$ $\mathcal{L}^{-1}\left[\dfrac{1}{s^2+a^2}\right] = \dfrac{1}{a}\sin at$

$\mathcal{L}[\cos at] = \dfrac{s}{s^2+a^2}$ $\qquad$ $\mathcal{L}^{-1}\left[\dfrac{s}{s^2+a^2}\right] = \cos at$

First Differentiation Formula

$$\mathcal{L}[f^{(n)}(t)] = s^n\,\mathcal{L}[f(t)] - s^{n-1}f(0) - s^{n-2}f'(0) - \dots - f^{(n-1)}(0)$$

$\mathcal{L}\left[\int_0^t f(u)\,du\right] = \dfrac{1}{s}\mathcal{L}[f(t)]$ $\qquad$ $\mathcal{L}^{-1}\left[\dfrac{1}{s}F(s)\right] = \int_0^t \mathcal{L}^{-1}[F(s)]\,du$

In the following formulas, $F(s) = \mathcal{L}[f(t)]$ so $f(t) = \mathcal{L}^{-1}[F(s)]$.

First Shift Formula

$\mathcal{L}[e^{at}f(t)] = F(s-a)$ $\qquad$ $\mathcal{L}^{-1}[F(s)] = e^{at}\,\mathcal{L}^{-1}[F(s+a)]$

Second Differentiation Formula

$\mathcal{L}[t^n f(t)] = (-1)^n \dfrac{d^n}{ds^n}\mathcal{L}[f(t)]$ $\qquad$ $\mathcal{L}^{-1}\left[\dfrac{d^nF(s)}{ds^n}\right] = (-1)^n t^n f(t)$

Second Shift Formula

$\mathcal{L}[u_a(t)g(t)] = e^{-as}\,\mathcal{L}[g(t+a)]$ $\qquad$ $\mathcal{L}^{-1}[e^{-as}F(s)] = u_a(t)f(t-a)$