1. (a) (i) Use Taylor's theorem to expand $\operatorname{Sin}\left(\frac{\pi}{4}+h\right)$ in ascending powers of h as far as the term in h^3.

(ii) Hence determine the value of Sin 47°, giving the answer correct to five decimal places.

(9 marks)

(b) (i) Determine the Maclaurin's series expansion for $f(x) = \ln(1 + 3x)$, in ascending powers of x up to and including x^4.

(ii) Hence evaluate $\int_0^1 \frac{1}{x}\ln(1+3x)dx$. (11 marks)

2. (a) Given the matrices

$$A = \begin{bmatrix} 1 & 2 & 1 \\ -2 & -1 & 2 \\ 1 & 3 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 1 & -1 & 1 \\ 3 & -1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

Find $A^2 - 4B$ (4 marks)

(b) Solve the equation:

$$\begin{vmatrix} x & 3 & 2 \\ 1 & 1 & x \\ x & 1 & 2 \end{vmatrix} = 4$$

(4 marks)

(c) Use inverse matrix method to solve the following simultaneous equations:

$$x + 2y + 3z = 6$$
$$2x + y + z = 5$$
$$3x + y - 2z = 1$$

(12 marks)

3. (a) Determine the inverse Laplace transform of $F(s) = \frac{2s^2 - 6s + 5}{s^3 - 6s^2 + 11s - 6}$. (8 marks)

(b) Use Laplace transforms to solve the differential equation:
$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 5e^t$, given that when t = o, x = 2 and $\frac{dx}{dt} = 1$. (12 marks)

4. (a) (i) A series circuit contains an inductor and resistor. The circuit is connected to a constant voltage source E at time t = 0. If initially the circuit was inactive, determine the expression for current for t > 0.

(ii) Hence obtain the expression for steady state current.

(8 marks)

(b) Use D - operator method to solve the following differential equation
$(D^2 + 4D + 3)y = x^3 + x^2 + 2$. (12 marks)

5. (a) Given that $u = \frac{x - y}{x + y}$

Show that $\frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} - \frac{2d^2u}{dxdy} = 0$ (7 marks)

(b) The power consumed in an electrical resistor is given by $P = \frac{E^2}{R}$ Watts, where E is the voltage drop across the resistor in Volts and R is the resistance of the resistor in ohms. Use partial derivatives to approximate change in power when E increases by 4 percent and R decreases by 0.05 percent, if the original values of E and R are 100 Volts and 10 ohms respectively. (7 marks)

(c) Determine the stationary values of $z = x^3 - 6xy + y^3$. (6 marks)

6. (a) (i) Given the vectors

$\underset{\sim}{A} = \underset{\sim}{i} + \underset{\sim}{j} + \underset{\sim}{k}$

$\underset{\sim}{B} = 2\underset{\sim}{i} + 3\underset{\sim}{j}$

$\underset{\sim}{C} = 4\underset{\sim}{i} + p\underset{\sim}{j} + 2\underset{\sim}{k}$

where p is a constant.
Find the value of p so that vectors $\underset{\sim}{A}$, $\underset{\sim}{B}$ and $\underset{\sim}{C}$ are co-planar.

(ii) Find a unit vector perpendicular to vectors $\underset{\sim}{M} = \underset{\sim}{i} + \underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{N} = \underset{\sim}{i} + 2\underset{\sim}{j} + 3\underset{\sim}{k}$. (7 marks)

(b) Given that $\phi(x,y,z) = 3x^2y + 4y^2z^3$. Determine the:

(i) directional derivative of ϕ in the direction of vector $\underset{\sim}{B} = 2\underset{\sim}{i} + 3\underset{\sim}{j} + 5\underset{\sim}{k}$ at the point (2,1,1).

(ii) Div. Grad ϕ at the point (3, -1, 2).

(8 marks)

(c) Vector $\underset{\sim}{A} = x^2y\underset{\sim}{i} + xyz^2\underset{\sim}{j} + x^3y\underset{\sim}{k}$. Determine curl $\underset{\sim}{A}$ at the point (2, 1, 2). (5 marks)

7. (a) Solve the differential equation $\frac{dy}{dx} + y \cot x = \sin x$. (5 marks)

(b) Use the method of undetermined coefficients to solve the differential equation
$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 6x + 4\cos x$, given that $y = 2$, $\frac{dy}{dx} = 4$ when $x = 0$. (15 marks)

8. (a) A radar unit is used to measure the speed of automobiles on an express way during rush hour traffic. The speeds of individual automobiles are normally distributed with a mean of 62 kph.

Determine the:

(i) standard deviation of all speeds if 3% of the automobiles travel faster than 72 kph.

(ii) Percentage of cars that travels with speeds less than 58 kph.

(iii) 95th percentile for the variable "speed".

(8 marks)

(b) The lifetime of an electrical bulb in years is represented by a continuous random variable T, defined by the probability density function,

$$f(t) = \begin{bmatrix} \frac{C^2}{2} e^{-ct} & ; & t \geq 0 \\ 0 & ; & \text{elsewhere} \end{bmatrix}$$

where c is a constant. Determine the:

(i) value of constant c;
(ii) expected lifetime;
(iii) median;
(iv) cumulative distribution function (c.d.f) F(t).

(12 marks)