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2305/301 2308/301
2306/301 2309/301
2307/301
MATHEMATICS
June/ July 2018
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN BUILDING
DIPLOMA IN QUANTITY SURVEYING
DIPLOMA IN CIVIL ENGINEERING
DIPLOMA IN HIGHWAY ENGINEERING
DIPLOMA IN ARCHITECTURE

MATHEMATICS

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:

Answer booklet;
Mathematical tables/ scientific calculator;
Drawing instruments.

*Answer any **FIVE** of the **EIGHT** questions.*
All questions carry equal marks.
Maximum marks for each part of a question are indicated.
Candidates should answer the questions in English.

This paper consists of 4 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

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1. (a) (i) Find the first four terms in the binomial expansion of $\left(1+\frac{1}{3}x\right)^{1/2}$.

(ii) By setting $x=\frac{1}{3}$ in the result in 1a(i), determine the value of $\sqrt{10}$ correct to four decimal places.

(7 marks)

(b) Given the complex numbers $Z_1 = 4+j2$, $Z_2 = 3+j2$ and $Z_3 = -3+j3$, determine $Z = \frac{Z_1 + Z_2 Z_3}{Z_2 + Z_3}$ leaving the answer in a + jb form. (6 marks)

(c) Given that $Z = 2 - j$ is one root of the polynomial $3Z^3 - 14Z^2 + 23Z + k = 0$, determine the:

(i) value of k;

(ii) other two roots.

(7 marks)

2. (a) A construction firm invests ksh 1.3 million on machinery in the first year. If the firm increases its investment on machinery by ksh 400,000 per year, determine the total amount spent at the end of the fifth year. (3 marks)

(b) Determine the area of the parallelogram whose sides are given by the vectors $\underset{\sim}{A} = 2\underset{\sim}{i} - 4\underset{\sim}{j} - \underset{\sim}{k}$ and $\underset{\sim}{B} = \underset{\sim}{i} + 2\underset{\sim}{j} + 6\underset{\sim}{k}$. (5 marks)

(c) Prove the identity $\frac{1+\sin 4\theta - \cos 4\theta}{\sin 2\theta} = 2(\sin 2\theta + \cos 2\theta)$. (4 marks)

(d) Given that $\sin(x+\theta) = 4\cos(x-\theta)$, where $\cos x \cos\theta \neq 0$:

(i) show that $\tan x = \frac{4\cos\theta - \sin\theta}{\cos\theta - 4\sin\theta}$.

(ii) Hence, solve the equation $\sin\left(x+\frac{\pi}{4}\right) = 4\cos\left(x-\frac{\pi}{4}\right)$ for values of x between 0° and 360° inclusive. (8 marks)

3. (a) If $y = 4e^{3x}\cos(2x-3)$, show that $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 13y = 0$. (5 marks)

(b) Given the parametric equations $x = 2(\cos\theta + \theta\sin\theta)$ and $y = 2(\sin\theta - \theta\cos\theta)$, determine $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{6}$. (7 marks)

(c) Determine the radius of curvature of the curve $y = \frac{10-3x}{2-x}$ at the point (1,7).

(8 marks)

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4. (a) Given the function $Z = y\tan^{-1}\left(\frac{x}{y}\right)$, show that $x\frac{dZ}{dx} + y\frac{dz}{dy} = Z$. (5 marks)

(b) The radius of cylinder increases at the rate of 0.3 cm/s while the height decreases at a rate of 0.5 cm/s. Use partial differentiation to determine the rate at which volume is changing at the instant when r = 7 cm and h = 10 cm. (6 marks)

(c) Locate the stationary points of the function $Z = 2x^2 + 2y^2 + 2xy + 12x - 12y + 9$ and determine their nature. (9 marks)

5. (a) Evaluate the integrals:

(i) $\int_0^{\pi} x^2 \sin x \, dx$;

(ii) $\int \frac{4x^2 + x + 1}{(x-1)(x^2+2)} dx$.

(12 marks)

(b) Determine the x coordinate of the centroid of the region bounded by the curve $y = 6\sin 2x$ the x axis and the straight line $x = 1$ and $x = \frac{\pi}{6}$. (8 marks)

6. (a) Show that the solution of the differential equation $x^2 - 6y^2 + 4xy\frac{dy}{dx} = 0$, satisfying the conditions $x = 1$, $y = 3$ may be expressed in the form $y = x\sqrt{\frac{17x+1}{2}}$.

(8 marks)

(b) Use the method of undetermined coefficients to solve the differential equation $9\frac{d^2x}{dt^2} - 6\frac{dx}{dt} + x = 9e^{\frac{1}{3}t}$. (12 marks)

7. (a) Given the matrices:

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 4 \\ 1 & 2 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 2 & 1 \\ 0 & 3 & 1 \\ 1 & 5 & 1 \end{bmatrix}$$

(i) find 6A - 3B;

(ii) determine $\det(AB)^T$.

(8 marks)

(b) Three forces F_1, F_2 and F_3 in newtons acting on a structural system satisfy the simultaneous equations:

$$3F_1 - 2F_2 + 4F_3 = 9$$
$$-6F_1 + 4F_2 + F_3 = 18$$
$$5F_1 + 3F_2 + 6F_3 = 25$$

Use Crammer's rule to determine the values of the forces. (12 marks)

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8. (a) **Table 1** shows marks scored by 50 students in a mathematics examination.

Table 1

Marks	10 - 19	20 - 29	30 - 39	40 - 49	50 - 59	60 - 69	70 - 79	80 - 89	90 - 100
No. of students (f)	3	5	7	8	10	7	5	4	1

Calculate the:

(i) median;
(ii) standard deviation of the data using an assumed mean of 54.5.

(10 marks)

(b) The diameter of a steel pipe used in construction is assumed to be a continuous random variable t with a probability density function.

$$f(t) = \begin{cases} k(1+2t)^2 & 0 \leq t \leq 1 \\ 0 & \textit{elsewhere} \end{cases}$$

Determine the:

(i) value of constant k;
(ii) mean;
(iii) $p(0.2 \leq t \leq 0.7)$.

(10 marks)

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