2521/201, 2602/203
2601/203, 2603/203
ENGINEERING MATHEMATICS II
June/July 2021
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

# DIPLOMA IN ELECTRICAL AND ELECTRONIC ENGINEERING (POWER OPTION) (TELECOMMUNICATION OPTION) (INSTRUMENTATION OPTION)

## MODULE II

ENGINEERING MATHEMATICS II

3 hours



## INSTRUCTIONS TO CANDIDATES

*You should have the following for this examination.*
*Answer booklet;*
*Scientific calculator;*
*Mathematical tables/Non-programmable scientific calculator.*
*Answer any **FIVE** of the following **EIGHT** questions in the answer booklet provided.*
*All questions carry equal marks.*
*Maximum marks for each part of a question are as indicated.*
***Candidates should answer the questions in English.***

**This paper consists of 6 printed pages.**

**Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.**

1. (a) Given the matrices

$$A=\begin{bmatrix}1 & 1 & 2\\ -1 & 1 & 1\\ 0 & -1 & 1\end{bmatrix} \quad \text{and} \quad B=\begin{bmatrix}1 & -1 & 1\\ 1 & 1 & -1\\ 1 & -2 & 1\end{bmatrix}$$

(i) Determine a matrix C = 2A - 3B.

(ii) Show that $(AB)^T = B^T A^T$.

(10 marks)

(b) Three currents $I_1$, $I_2$ and $I_3$ in amperes, flowing in a d.c network satisfy the simultaneous equations:

$$I_1 - 2I_2 + 3I_3 = 5$$

$$-2I_1 + I_2 + 2I_3 = 3$$

$$I_1 + I_2 - I_3 = 0$$

Use the inverse matrix method to determine the values of the currents. (10 marks)

2. (a) Show that the general solution of the differential equation

$$\frac{y}{x^2}\frac{dy}{dx} + \frac{1+y^2}{1+x^3} = 0$$

may be expressed in the form $(1+y^2)^3(1+x^3)^2 = c$, where $c$ is an arbitrary constant.

$y \ln(1+y^2) = x^2 \ln(1+x^3). + \ln c$ (7 marks)



(b) The charge $q(t)$ on the plates of a capacitor satisfies the differential equation

$$2\frac{d^2q}{dt^2} + 7\frac{dq}{dt} + 3q = \sin t$$

Use the method of undetermined coefficients to determine the general solution of the equation. (13 marks)

3. (a) Given $u = \tan^{-1}(y/x)$, show that

$$\frac{\partial^2 u}{dx^2} + \frac{\partial^2 u}{dy^2} = 0$$

(8 marks)

(b) Use partial differentiation to determine the equation of the tangent to the curve $x^2 - 3y^2 + 6xy - 2x + 6y$ at the point (1,1). (6 marks)

(c) Locate the stationary points of the function $f(x,y) = x^2 + xy + y^2$, and state their nature. (6 marks)



$0.5 - 3.5 + 3.$
$= 0.$
$13 \quad -21 + 3$

4. (a) Find the:

(i) Laplace transform of $f(t) = t\cos 4t$;

(ii) inverse Laplace transform of $F(s) = \dfrac{4s+1}{(2s-1)(s^2+4)}$

(9 marks)

(b) The circuit in Figure 1 is dead prior to switch closure at $t = 0$.

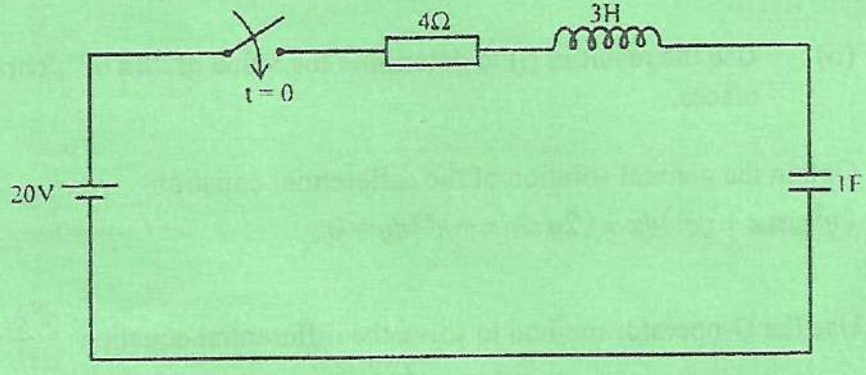


Fig. 1

Use Laplace transforms to determine an expression for the charge $q(t)$ on the capacitor, for $t \geq 0$. (11 marks)

5. (a) Given the vectors $\underset{\sim}{A} = 4\underset{\sim}{i} + 3\underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{B} = 2\underset{\sim}{i} + \underset{\sim}{j} - 2\underset{\sim}{k}$, determine:



(i) a unit vector perpendicular to $\underset{\sim}{A}$ and $\underset{\sim}{B}$;

(ii) the angle between $\underset{\sim}{A}$ and $\underset{\sim}{B}$.

(10 marks)

(b) A scalar field is given by $\phi = x^3y^2 + 2xz^2$. Determine, at the point (–1, 1, 2):

x y z

(i) $\nabla\phi$

(ii) the directional derivative of $\phi$ in the direction of the vector $\underset{\sim}{A} = 3\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}$.

(6 marks)

(c) A vector field $\underset{\sim}{E} = -yx\underset{\sim}{i} + yz^2\underset{\sim}{j} + zx\underset{\sim}{k}$ exists in a region of space. Determine $\nabla \times \underset{\sim}{E}$ at the point (1, 2, 2). (4 marks)

$x = -2y.$

$2x + y = 0.$

$2(-2y) + y = 0.$

$-4y + y = 0.$

$3y = 0.$

6. (a) (i) Determine the first three non-zero terms in the Maclaurin series expansion of $f(x) = \sin 2x$.

(ii) Hence, evaluate the integral

$\int_0^1 \frac{\sin 2x}{\sqrt{x}}\, dx$, correct to three decimal places. (10 marks)

(b) (i) Use Taylor's theorem to expand $\tan(\pi/3 + h)$ as far as the term in $h^3$.

(ii) Use the result in (i) to determine the value of $\tan 63°$, correct to three decimal places.

(10 marks)

7. (a) Obtain the general solution of the differential equation

$(y^2 \cos x + ye^x)dx + (2y \sin x + e^x)dy = 0$. (7 marks)

(b) Use the D-operator method to solve the differential equation $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = xe^{-x}$, given that when $x = 0, y = 0$ and $\frac{dy}{dx} = 0$. (13 marks)

8. (a) A continuous random variable $x$ is normally distributed with mean $\mu$ and standard deviation $\sigma$. The probability that $x$ is greater than 8 is 0.7 and the probability that $x$ is less than 4 is 0.2. Determine the values of:

(i) $\sigma$

(ii) $\mu$

(9 marks)

$y = ke^{x}$ $y = kxe^{x}$

(b) A sample of 25 bolts have masses which are normally distributed. If the mean and standard deviation are 40.0 g and 1.5 g respectively, determine the:

(i) unbiased point estimate of the standard deviation;

(ii) 95% confidence interval of the mean.

(11 marks)

$e^{at}(A \sin t + B \sin t)$.





201-3
202-1
203-2
204-1
205-(5)