2521/303
2601/303
2602/303
2603/303
ENGINEERING MATHEMATICS III
June/July 2021
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN ELECTRICAL AND ELECTRONIC ENGINEERING
(POWER OPTION)
(TELECOMMUNICATION OPTION)
(INSTRUMENTATION OPTION)

MODULE III

ENGINEERING MATHEMATICS III

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:
Mathematical tables/ Scientific calculator;
Answer booklet.
*Answer any **FIVE** of the following **EIGHT** questions in the answer booklet provided.*
***ALL** questions carry equal marks.*
Maximum marks for each part of a question are as indicated.
Candidates should answer the questions in English.

This paper consists of 6 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

1. (a) (i) Using Newton-Raphson method show that a better approximation to the root of the equation $x^3 - 3x - 5 = 0$ is given by

$$x_{n+1} = \frac{2x_n^3 + 5}{3x_n^2 - 3}, n = 0, 1, 2, ...$$

(ii) Hence determine the root of the equation by taking the first approximation as $x_0 = 2$.
Give your answer correct to three decimal places.

(9 marks)

(b) Table 1 represents a polynomial of degree 3, f(x).

Table 1

x	1	1.4	1.8	2.2
f(x)	3.49	4.82	5.96	6.5

Use Newton-Gregory interpolation formula to determine:

(i) f(1.2);

(ii) f(2.0).

correct to 4 decimal places.

(11 marks)

2. (a) Given that $u(x,y) = 3x^2y + 2x^2 - y^3 - 2y^2$.

(i) show that $u(x,y)$ is harmonic;

(ii) determine a harmonic conjugate function $v(x,y)$ such that $u(x,y)$ and $v(x,y)$ satisfies Cauchy-Riemann equations;

(iii) find $f(z)$ and $f'(z)$ in terms of z.

(12 marks)

(b) The circle $|z| = 2$ is mapped onto the w-plane under the transformation

$$w = \frac{1}{z - 3j}$$

Determine the:

(i) centre;
(ii) radius of the image circle.

(8 marks)

3. (a) Determine the eigenvalues and the corresponding eigenvectors of the matrix

$$A=\begin{bmatrix}1 & 1\\ 9 & 1\end{bmatrix}$$

(10 marks)

(b) A system of linear differential equations with constant coefficients can be written in vector-matrix form as:

$$\frac{dx(t)}{dt}=Bx(t) \quad \text{where} \quad B=\begin{bmatrix}0 & 1\\ 8 & -2\end{bmatrix}.$$

Determine the state transition matrix $\phi(t)$ of the system.

(10 marks)

4. (a) (i) Sketch the region of integration of the integral.

$$\int_0^{\sqrt{2}}\int_{\sqrt{(4-2y^2)}}^{\sqrt{(4-2y^2)}} y\,dx\,dy$$

(ii) Evaluate the integral in a(i).

(6 marks)

(b) Change the order of integration and hence evaluate the integral.

$$\int_0^1\int_{\sqrt{x}}^1 \frac{y^2\,dy\,dx}{\sqrt{(y^4-x^2)}}$$

(7 marks)

(c) Determine the triple integral

$\iiint_v x^2\,dz\,dy\,dx$ where v is the volume bounded by the surfaces.

$x^2+y^2=9$, $z=0$ and $z=2$.

(7 marks)

5. (a) Verify Green's theorem in the plane for $\oint_c [(xy+y^2)dx+x^2dy]$ where c is the curve of region $y = x^2$ and $y = x$.

(11 marks)

(b) Show that

$\int_{(1,2)}^{(3,4)}[(6xy^2-y^3)dx+(6x^2y-3xy^2)dy]$ is independent of path joining (1,2) and (3,4).

Hence evaluate it from (1,2) to (3,2) and from (3,2) to (3,4).

(9 marks)

6. (a) (i) Sketch an even extension of the function

$$f(x)=\begin{cases} -x+1, & 0<x<1 \\ x-1, & 1<x<2 \\ f(x+4) & \end{cases}$$

in the interval $-2<x<2$.

(ii) Determine Fourier series representation of the function in (i). (11 marks)

(b) A function g(t) is defined by

$$g(t)=\begin{cases} t^2, & -\pi<t<\pi \\ g(t+2\pi) & \end{cases}$$

Sketch the function in the interval $-\pi<t<3\pi$ and determine its Fourier series. (9 marks)

7. (a) Evaluate the triple integral

$$\int_0^1\int_0^{1-x}\int_0^{2-x} xyz\,dz\,dy\,dx.$$

(7 marks)

(b) Use Divergence theorem to evaluate

$$\iint_S \left(y^2z^2\underset{\sim}{i}+z^2x^2\underset{\sim}{j}+z^2y^2\underset{\sim}{k}\right).\widehat{n}\,dS$$

where S is the part of the sphere $x^2+y^2+z^2=1$ above the xy - plane and bounded by this plane. (8 marks)

(c) Evaluate $\oint_C [x\,dy-y\,dx]$ along the circle $x^2+y^2=1$. (5 marks)

8. (a) By changing to polar co-ordinates, evaluate:

$$\iint_D \frac{x^2}{(x^2+y^2)^{\frac{3}{2}}}dA \text{ where}$$

D is the region bounded by the circles $x^2+y^2=a^2$, $x^2+y^2=b^2$, $0<a<b$ and a and b are constants. (6 marks)

(b) Verify stoke's theorem for

$\underset{\sim}{F}=xz\underset{\sim}{i}-y\underset{\sim}{j}+x^2y\underset{\sim}{k}$, where S is the surface of the region bounded by

$x=0, y=0, z=0, 2x+y+2z=8$ which is not included in the xz - plane.

(14 marks)

TABLE OF LAPLACE TRANSFORM FORMULAS

$$\mathcal{L}[t^n] = \frac{n!}{s^{n+1}} \qquad \mathcal{L}^{-1}\left[\frac{1}{s^n}\right] = \frac{1}{(n-1)!}t^{n-1}$$

$$\mathcal{L}[e^{at}] = \frac{1}{s-a} \qquad \mathcal{L}^{-1}\left[\frac{1}{s-a}\right] = e^{at}$$

$$\mathcal{L}[\sin at] = \frac{a}{s^2+a^2} \qquad \mathcal{L}^{-1}\left[\frac{1}{s^2+a^2}\right] = \frac{1}{a}\sin at$$

$$\mathcal{L}[\cos at] = \frac{s}{s^2+a^2} \qquad \mathcal{L}^{-1}\left[\frac{s}{s^2+a^2}\right] = \cos at$$

First Differentiation Formula

$$\mathcal{L}[f^{(n)}(t)] = s^n\mathcal{L}[f(t)] - s^{n-1}f(0) - s^{n-2}f'(0) - \ldots - f^{(n-1)}(0)$$

$$\mathcal{L}\left[\int_0^t f(u)\,du\right] = \frac{1}{s}\mathcal{L}[f(t)] \qquad \mathcal{L}^{-1}\left[\frac{1}{s}F(s)\right] = \int_0^t \mathcal{L}^{-1}[F(s)]\,du$$

In the following formulas, $F(s) = \mathcal{L}[f(t)]$ so $f(t) = \mathcal{L}^{-1}[F(s)]$.

First Shift Formula

$$\mathcal{L}[e^{at}f(t)] = F(s-a) \qquad \mathcal{L}^{-1}[F(s)] = e^{at}\mathcal{L}^{-1}[F(s+a)]$$

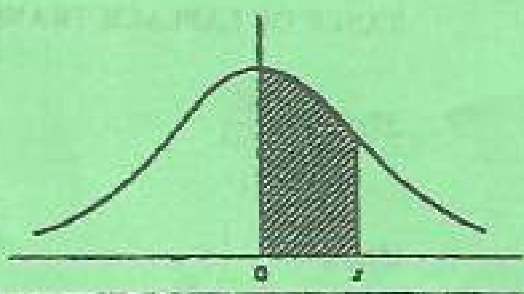
Second Differentiation Formula

$$\mathcal{L}[t^n f(t)] = (-1)^n\frac{d^n}{ds^n}\mathcal{L}[f(t)] \qquad \mathcal{L}^{-1}\left[\frac{d^nF(s)}{ds^n}\right] = (-1)^n t^n f(t)$$

Second Shift Formula

$$\mathcal{L}[u_a(t)g(t)] = e^{-as}\mathcal{L}[g(t+a)] \qquad \mathcal{L}^{-1}[e^{-as}F(s)] = u_a(t)f(t-a)$$

Partial areas under the standardised normal curve



$z = \frac{x-\bar{x}}{\sigma}$	0	1	2	3	4	5	6	7	8	9
0.0	0.0000	0.0040	0.0080	0.0120	0.0159	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2760	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4430	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4762	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4980	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000

THIS IS THE LAST PRINTED PAGE.