2521/201, 2602/203
2601/203, 2603/203
ENGINEERING MATHEMATICS II
June/July 2022
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN ELECTRICAL AND ELECTRONIC ENGINEERING
(POWER OPTION)
(TELECOMMUNICATION OPTION)
(INSTRUMENTATION OPTION)

MODULE II

ENGINEERING MATHEMATICS II

3 hours

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INSTRUCTIONS TO CANDIDATES

You should have the following for this examination.
- *Drawing instruments;*
- *Mathematical tables/Non-programmable scientific calculator;*
- *Answer booklet;*
- *Abridged tables of Laplace transforms and standard normal distribution.*

*This paper consists of **EIGHT** questions.*
*Answer any **FIVE** questions in the answer booklet provided*
All questions carry equal marks.
Maximum marks for each part of a question are as indicated.
Candidates should answer the questions in English.

This paper consists of 6 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

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Turn over

1. (a) Determine the unit tangent vector to the curve whose parametric equations are: $x = 3\cos t, y = 3\sin t$ and $z = 4t$. (4 marks)

(b) Given the two surfaces $P = 2x^2 + 3y^2 + 6z^2 + 8$, $Q = 4x^2y + 3xy^2z^2 + 2xyz$, determine at the point (1, 2, 1)

(i) unit normals to the surfaces;
(ii) the angle between the surfaces.

(8 marks)

(c) If vector $\underset{\sim}{F}$ and scalar Q are given by $\underset{\sim}{F} = 2x^2yz\underset{\sim}{i} + 4xyz\underset{\sim}{j} - 2xyz^2\underset{\sim}{k}$, $Q = 4x^2y^3z^4$ respectively. Determine at the point (1, –1, 2)

(i) $\nabla\phi$

(ii) $\nabla\cdot\underset{\sim}{F}$

(iii) $\nabla\times\underset{\sim}{F}$

(8 marks)

2. (a) (i) Determine the first four non-zero terms of the Maclaurin's series expansion of $f(x) = e^{\sin x}$.

(ii) Hence, evaluate the integral $\int_1^2 \frac{e^{\sin x}}{x}\,dx$.

(10 marks)

(b) (i) Use Taylors' theorem to expand

$f(x) = \ln[\sin(x+h)]$ to four terms.

(ii) Hence determine the value of $\ln\sin 31^\circ$.

(10 marks)

3. (a) Given that $w = e^{6x}\cos(6y+8) + 8y + 9x + 10$ determine $\frac{\partial^2 w}{dx^2} + \frac{\partial^2 w}{dy^2}$. (5 marks)

(b) A rectangular box, open at the top, has a volume of 32.0 cm³. Determine, using partial differentiation, the dimensions of the box requiring least material for construction.

(15 marks)

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4. (a) Given the matrices

$$A=\begin{bmatrix}1 & 2 & 3\\ 1 & -2 & 4\\ 6 & 3 & 5\end{bmatrix} \text{ and } B=\begin{bmatrix}5 & 4 & 3\\ 1 & -5 & 2\\ 6 & 2 & 8\end{bmatrix}$$

determine $(A^TB) - 3A + 2B$. (7 marks)

(b) **Figure 1** shows a three-loop d.c network

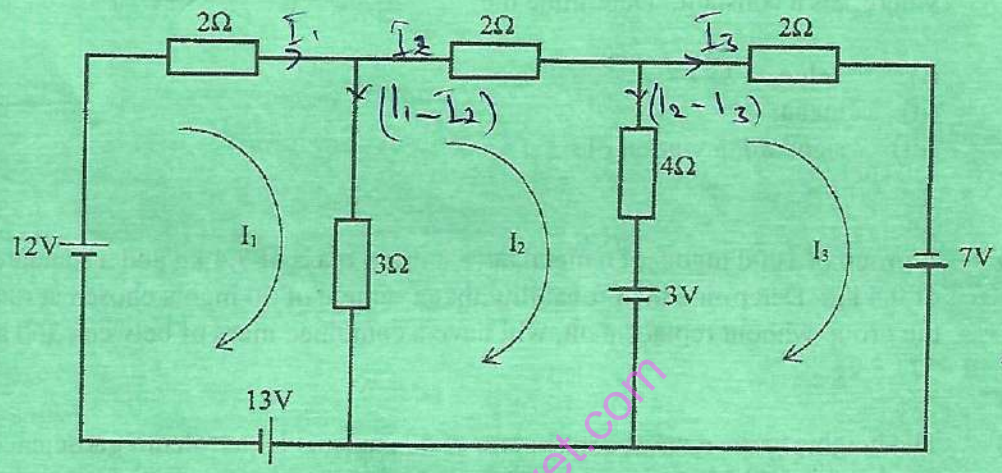

Fig. 1

Use Cramer's rule to determine the values of the currents I_1, I_2 and I_3. (13 marks)

5. (a) Find the Laplace transform of $te^{4t}\cos 3t$ from first principles. (10 marks)

(b) Use Laplace transforms to solve the differential equation

$$\frac{d^2x}{dt^2}-4\frac{dx}{dt}+4x=te^{-2t}$$

given that when $t=0$, $x=1$ and $\frac{dx}{dt}=-5$. (10 marks)

6. (a) Solve the differential equation:

$$\frac{dy}{dx}=\frac{12x^2+9xy+y^2}{x^2}$$ (7 marks)

(b) Use the method of undetermined co-efficients to solve the differential equation:

$\frac{d^2x}{dt^2}+4\frac{dx}{dt}+5x=4\sin 3t$, given that when $t=0$, $x=1$ and $\frac{dx}{dt}=3$. (13 marks)

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7. (a) A sample of people is randomly selected and tested for Covid-19 from a population of 625 people. It is established that they have a mean of 20 and a standard deviation of 2.56. Determine the probability that 3 people are confirmed to suffer from the corona virus. (8 marks)

(b) A continuous random variable x has a probability density function $f(x)$ defined by

$$f(x)=\begin{cases} k(x^2+2x) & 1\leq x\leq 3 \\ 0, \quad elsewhere \end{cases}$$

where k is a constant. Determine the:

(i) value of k
(ii) mean;
(iii) standard deviation of x.

(12 marks)

8. (a) A group of 1000 ingots of a metal have a mean mass of 7.4 kg and a standard deviation of 0.4 kg. Determine the probability that a sample of 50 ingots chosen at random from the group without replacement, will have a combined mass of between 360 and 377.5 kg. (7 marks)

(b) The heights to the nearest centrimetre of 50 students in petroleum geoscience class are tabulated in table 1.

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Table 1

165	160	158	172	186	165	158	142	140	180
180	184	156	159	167	175	180	165	138	190
174	170	168	155	148	168	175	182	180	145
135	184	182	140	147	182	158	186	175	150
156	148	174	169	148	143	163	188	180	175

(i) Make a frequency table of interval 10 e.g 130 - 140, ---.

(ii) Hence determine the co-efficient of skewness.

(13 marks)

TABLE OF LAPLACE TRANSFORMS

	FUNCTION F(t)	TRANSFORM $\int_0^\infty e^{-st} F(t)\,dt$
1.	1	$1/s$
2.	e^{at}	$1/(s - a)$
3.	sin at	$a/(s^2 + a^2)$
4.	cos at	$s/(s^2 + a^2)$
5.	t	$1/s^2$
6.	t^n (n a +ve integer)	$n!/s^{n+1}$
7.	sinh at	$a/(s^2 - a^2)$
8.	cosh at	$s/s^2 - a^2)$
9.	t sin at	$2as/(s^2 + a^2)^2$
10.	t cos at	$(s^2 - a^2)/(s^2 + a^2)^2$
11.	$e^{-at}t^n$	$n!/(s + a)^{n+1}$
12.	e^{-at} cos ωt	$(s + a)/[(s + a)^2 + \omega^2]$
13.	e^{-at} sin ωt	$\omega/[(s + a)^2 + \omega^2]$
14.	e^{-at} cosh ωt	$(s + a)/[(s + a)^2 - \omega^2]$
15.	e^{-at} sinhωt	$\omega/[(s + a)^2 - \omega^2]$

Some Theorems used in Laplace Transforms.

1. If $f(s) = L\{F(t)\}$, then $f(s + a) = L\{e^{-at} F(t)\}$
2. $L\{dx/dt\} = sL\{x\} - x(0)$ (b) $L\{d^2x/dt^2\} = s^2L\{x\} - sx(0) - x'(0)$

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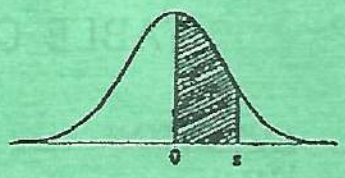
AREAS under the STANDARD NORMAL CURVE from 0 to z



z	0	1	2	3	4	5	6	7	8	9
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0754
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2147	0.2190	0.2224
0.6	0.2258	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2518	0.2549
0.7	0.2580	0.2612	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2996	0.3023	0.3051	0.3078	0.3206	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000

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