2521/201 2602/203
2601/203 2603/203
ENGINEERING MATHEMATICS II
Oct./Nov. 2022
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN ELECTRICAL AND ELECTRONIC ENGINEERING (POWER OPTION) (TELECOMMUNICATION OPTION) (INSTRUMENTATION OPTION)

MODULE II

ENGINEERING MATHEMATICS II

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:
- *Answer booklet;*
- *Drawing instruments;*
- *Mathematical tables/Non-programmable scientific calculator.*

*This paper consists **EIGHT** questions.*
*Answer any **FIVE** questions in the answer booklet provided.*
All questions carry equal marks.
Maximum marks for each part of a question are as indicated.
Candidates should answer the questions in English.

This paper consists of 7 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

1. (a) Given that $A = \begin{bmatrix} 2 & 4 & 1 \\ 3 & 5 & 7 \\ 2 & 8 & 2 \end{bmatrix}$ $and\ B = \begin{bmatrix} -1 & 2 & 3 \\ 1 & 5 & 7 \\ 1 & 2 & 3 \end{bmatrix}$,

Determine:

(i) $A+B^T$

(ii) $(AB)^{-1}$

(8 marks)

(b) Given the matrix $Q = \begin{bmatrix} 12 & 12 & -6 \\ -6 & 12 & 12 \\ 12 & -6 & 12 \end{bmatrix}$

(i) If $QQ^T = \lambda I$ where I is an identity matrix, determine;

I. λ

II. Q^{-1}.

(ii) Hence solve the equations:

$$12I_1 + 12I_2 - 6I_3 = -18$$

$$-6I_1 + 12I_2 + 12I_3 = 0$$

$$12I_1 - 6I_3 + 12I_3 = 72$$

(12 marks)

2. (a) Determine the:

(i) Laplace transform of $f(t) = e^{3t}(\cos 3t \sin 4t)$;

(ii) Inverse Laplace transform of

$$F(s) = \left[\frac{2s+7}{(s+2)(s^2+6s+10)}\right].$$

(10 marks)

(b) Use Laplace transforms to solve the simultaneous differential equations:

$$\frac{dy}{dt} + 2x = \sin 2t,$$

$$\frac{dx}{dt} - 2y = \cos 2t$$

given that when $t = 0, x = 1$ and $y = 0$.

(10 marks)

3. (a) Solve the differential equation

$$x \cos x \frac{dy}{dx} + (x \sin x + \cos x)y = 1.$$

(7 marks)

(b) The differential equation describing the variation of a capacitor charge q in an alternating current circuit containing inductance L, resistance R and capacitance C in series is given by:

$$L\frac{d^2q}{dt^2}+R\frac{dq}{dp}+\frac{1}{c}q=V_0\sin\omega t$$

Use the method of undetermined coefficients to determine the charge q on the capacitor in the circuit at any time t seconds given that when $t=0$, $q=0$ and $\frac{dq}{dt}=-44.29$, $R=20$ Ohms, $L=10\times10^{-3}$ Henry, $C=100\times10^{-6}$ Farads, $V_0=500$ Volts and $\omega=250$. (13 marks)

4. (a) Determine the angle between the vectors $\underset{\sim}{A}=4\underset{\sim}{i}-3\underset{\sim}{j}+7\underset{\sim}{k}$ and $\underset{\sim}{B}=2\underset{\sim}{i}+7\underset{\sim}{j}+4\underset{\sim}{k}$. (6 marks)

(b) Given $\phi=x^2yz-4y^2z-3$, determine the directional derivative of ϕ at the point $(1,-1,2)$ in the direction of the vector $\underset{\sim}{r}=5\underset{\sim}{i}+4\underset{\sim}{j}+10\underset{\sim}{k}$. (6 marks)

(c) The acceleration of a particle at any time $t\geq0$ is given by:

$$\underset{\sim}{a}=e^{-t}\underset{\sim}{i}-6(t+1)\underset{\sim}{j}+3\sin t\underset{\sim}{k}$$

If both the velocity $\underset{\sim}{V}$ and displacement $\underset{\sim}{S}$ are zero at time $t\geq0$, determine expressions for $\underset{\sim}{V}(t)$ and $\underset{\sim}{S}(t)$. (8 marks)

5. (a) If $V=(x^2+y^2+z^2)^{-\frac{1}{2}}$ show that $\frac{\partial^2V}{\partial x^2}+\frac{\partial^2V}{\partial y^2}+\frac{\partial V^2}{\partial z^2}=0$. (5 marks)

(b) If $u=\sin^{-1}\left(\frac{x}{y}\right)+\tan^{-1}\left(\frac{y}{x}\right)$, determine $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}$. (7 marks)

(c) Locate the stationary points of the function $f(x,y)=x^2+4y^2-5xy-29x+50y+17$ and determine their nature. (8 marks)

6. (a) Determine the first four non-zero terms of the Maclaurin's series expansion of $\tan^{-1}x^2$. (10 marks)

(b) (i) Expand $x^{\frac{1}{2}}$ in Taylor's series about $x=a$ in powers of h up to the fourth term.

(ii) Hence determine $\sqrt{10}$. (10 marks)

7. (a) A company has a number of production lines of which an average of 4 are in use at any instant. Assuming that the number of lines in use at any instant follows a Poisson distribution, determine the probability that:

(i) none;

(ii) exactly two;

(iii) at most three;

(iv) at least four

lines are in use.

(8 marks)

(b) A continuous random variable x has a probability density function $f(x)$ defined by

$$f(x) = \begin{cases} k(9x - 18 - x^2) \\ 0, & elsewhere \end{cases}$$

where k is a constant.

Determine the:

(i) Value of constant k;

(ii) mean;

(iii) standard deviation.

(12 marks)

8. (a) The masses and heights of 10 electrical engineering students are measured and their results tabulated in table 1.

Table 1

Mass (kg)	38	38	38	44	44	51	32	51	77	32
Height (cm)	135	140	137	141	147	145	132	149	164	130

Calculate the coefficient of correlation and comment on it. (10 marks)

(b) The power needed to drive a lathe machine as the cutting angle of the tool increases at constant speed and the depth of the cut. The relationship for the mild steel is shown in table 2.

Table 2

Cutting angle in degrees	50	55	60	65	70	75	80	85	90
Power (kW)	6.2	6.8	7.6	8.2	8.1	8.8	9.7	10.0	10.4

Determine:

(i) the regression line of power on cutting angle;

(ii) power when the angle is $30°$.

(10 marks)