2521/201, 2602/203
2601/203, 2603/203
ENGINEERING MATHEMATICS II
June/July 2019
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN ELECTRICAL AND ELECTRONIC ENGINEERING
(POWER OPTION)
(TELECOMMUNICATION OPTION)
(INSTRUMENTATION OPTION)

MODULE II

ENGINEERING MATHEMATICS II

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination.
    Answer booklet;
    Scientific calculator;
    An abridged table of laplace transform standard normal table.
Answer any FIVE of the following EIGHT questions in the answer booklet provided.
All questions carry equal marks.
Maximum marks for each part of a question are as indicated.
Candidates should answer the questions in English.

This paper consists of 6 printed pages.

Candidates should check the question paper to ascertain that
all the pages are printed as indicated and that no questions are missing.

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1. (a) Given the matrix $A = \begin{bmatrix} 2 & 3 & 5 \\ 4 & 1 & 6 \\ 1 & 4 & 0 \end{bmatrix}$, determine $A^{-1}A^{\mathsf{T}}$. (8 marks)

(b) Three currents I_1, I_2 and I_3 in ampheres flowing in an electrical circuit satisfy the simultaneous equation.

$$I_1 + I_2 + I_3 = 4$$
$$2I_1 - 3I_2 + 4I_3 = 33$$
$$3I_1 - 2I_2 - 2I_3 = 2$$

Use Cramer's rule to determine the value of the currents. (12 marks)

2. (a) Given that $z = \cos\left(\dfrac{y}{x}\right)$, show that

$$x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} = 0$$ (7 marks)

(b) If $z = 3x^4 \cos 2y$, use partial differentiation to determine the rate of change of z, if x increases by 4 units/s and y decreases by 2 units/s, at the instant when $x = 3$ units and $y = \dfrac{\pi}{6}$ radians. (5 marks)

(c) Locate the stationary points of the function $z = 2x^2 + y^2 - 2x - 2y + 2xy - 4$ and determine their nature. (8 marks)

3. (a) Show that the solution of the differential equation $x\dfrac{dy}{dx} = \dfrac{2}{x+2} - y$, given that when $x = -1$, $y = 0$, may be expressed in the form $y = \dfrac{2}{x}\ln(x+2)$. (7 marks)

(b) The equation of motion of a body undergoing damped forced vibrations is given by

$$\frac{d^2x}{dt^2} - 4\frac{dx}{dt} + 4x = 5\cos t.$$

Use the method of undetermined coefficients to solve the differential equation given that when $t = 0$, $x = 1$ and $\dfrac{dx}{dt} = 2$. (13 marks)

4. (a) Find the:

(i) laplace transform of $f(t) = \dfrac{\sin^2 3t}{t}$;

(ii) inverse laplace transform of $f(s) = \dfrac{4s - 15}{s^2 - 6s + 13}$. (9 marks)

(b)　Use laplace transforms to determine the current $i_2(t)$ in the network of Figure 1 assuming that the circuit is dead at $t=0$.
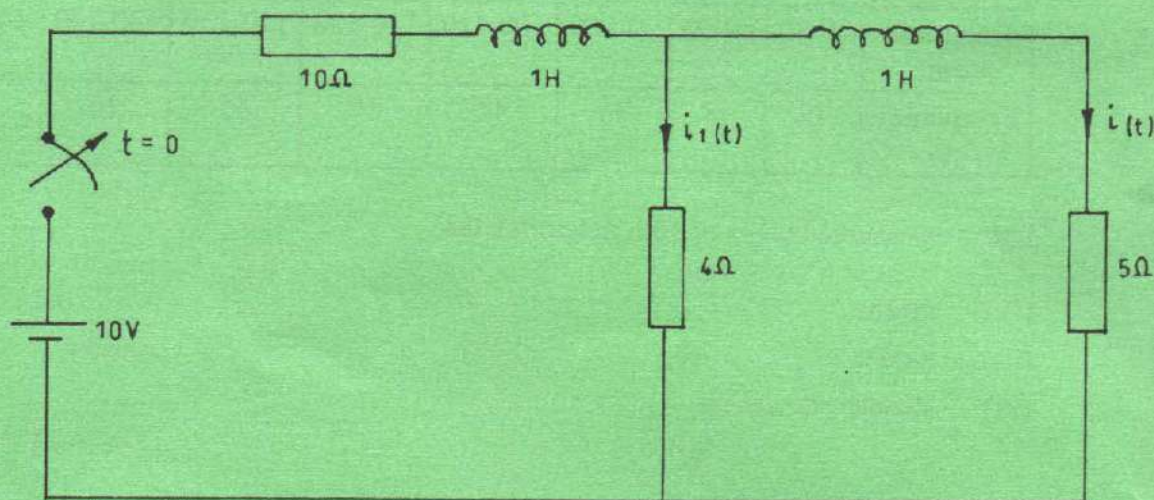

Fig. 1

(11 marks)

5.　(a)　Find the first four non-zero terms in the Maclaurin's series expansion of $f(x)=\cosh 2x$ and hence evaluate $\int_0^1 \dfrac{\cosh 2x}{x^{\frac{1}{3}}}\,dx$, correct to four decimal places.　(12 marks)

(b)　(i)　Use Taylor's theorem to expand $\sin\left(\dfrac{\pi}{3}+h\right)$ as far as the term in h^4.

(ii)　Hence, evaluate $\sin 64°$, correct to three decimal places.

(8 marks)

6.　(a)　Determine the area of the parallelogram whose sides are given by the vectors:

$A=2i-4j+5k$ and $B=2i+3j-4k$.　(6 marks)

(b)　Determine the directional derivative of the scalar field $\phi(x,y,z)=x^2-2y^2+2z^2$ in the direction of the vector $A=4i-2j+k$, at the point $(1,2,1)$.　(7 marks)

(c)　Given the magnetic field vector

$$A=(3x^2-3yz)i+(3y^2-3zx)j+(3z^2-3xy)k,$$

determine, at the point $(1,1,2)$

(i)　$\nabla \cdot A$

(ii)　$\nabla \times A$

(7 marks)

7. (a) Table 1 shows the lengths in centimetres of 230 cables used in an electrical workshop.

Table 1

Length in cm	70-80	80-90	90-100	100-110	110-120	120-130	130-140	140-150
Frequency (f)	12	18	35	42	50	45	20	8

Using an assumed mean of 105, determine the:

(i) mean;

(ii) mode;

(iii) standard deviation.

(11 marks)

(b) Table 2 shows marks scored by 10 students in digital electronic and mathematics tests.

Table 2

Marks scored in digital electronics	78	36	25	98	75	82	90	65	62	39
Marks scored in Mathematics	84	51	60	91	68	62	86	53	58	47

Calculate the Karl Pearson's correlation coefficient. (9 marks)

8. (a) The mean mass of a set of components is 700 grammes and the standard deviation is 45 grammes. Assuming the masses are normally distributed about the mean, determine for a sample of 42 components, how many are likely to have masses of:

(i) less than 650 grammes;

(ii) between 630 and 720 grammes.

(9 marks)

(b) A continuous random variable x has a probability density function $f(x)$ defined by:

$$f(x) = \begin{cases} 3x(k-x) & 0 \leq x \leq 1 \\ 0 & \textit{elsewhere} \end{cases}$$

Determine the:

(i) value of the constant k;

(ii) mean;

(iii) variance.

(11 marks)

TABLE OF LAPLACE TRANSFORMS

FUNCTION $F(t)$	**TRANSFORM** $\int_0^\infty e^{-st} F(t)\, dt$
1. 1	$1/s$
2. e^{at}	$1/(s-a)$
3. $\sin at$	$a/(s^2 + a^2)$
4. $\cos at$	$s/(s^2 + a^2)$
5. t	$1/s^2$
6. t^n (n a +ve integer)	$n!/s^{n+1}$
7. $\sinh at$	$a/(s^2 - a^2)$
8. $\cosh at$	$s/(s^2 - a^2)$
9. $t \sin at$	$2as/(s^2 + a^2)^2$
10. $t \cos at$	$(s^2 - a^2)/(s^2 + a^2)^2$
11. $e^{-at} t^n$	$n!/(s + a)^{n+1}$
12. $e^{-at} \cos \omega t$	$(s + a)/[(s + a)^2 + \omega^2]$
13. $e^{-at} \sin \omega t$	$\omega/[(s + a)^2 + \omega^2]$
14. $e^{-at} \cosh \omega t$	$(s + a)/[(s + a)^2 - \omega^2]$
15. $e^{-at} \sinh \omega t$	$\omega/[(s + a)^2 - \omega^2]$

Some Theorems used in Laplace Transforms.

1. If $f(s) = L\{F(t)\}$, then $f(s + a) = L\{e^{-at} F(t)\}$
2. $L\{dx/dt\} = sL\{x\} - x(0)$ (b) $L\{d^2x/dt^2\} = s^2 L\{x\} - sx(0) - x'(0)$

AREAS
under the
STANDARD
NORMAL CURVE
from 0 to z
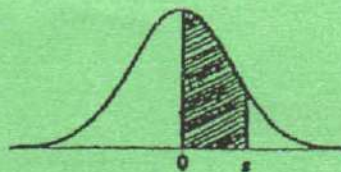
z	0	1	2	3	4	5	6	7	8	9
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0754
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2147	0.2190	0.2224
0.6	0.2258	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2518	0.2549
0.7	0.2580	0.2612	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2996	0.3023	0.3051	0.3078	0.3206	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000

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