2521/303 2602/303
2601/303 2603/303
ENGINEERING MATHEMATICS III
June/July 2019
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN ELECTRICAL AND ELECTRONIC ENGINEERING
(POWER OPTION)
(TELECOMMUNICATION OPTION)
(INSTRUMENTATION OPTION)

MODULE III

ENGINEERING MATHEMATICS III

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:
Answer booklet;
Mathematical tables/Non-programmable scientific calculator.
*This paper consists of **EIGHT** questions.*
*Answer any **FIVE** questions.*
Maximum marks for each part of a question are as indicated.
Candidates should answer the questions in English.

This paper consists of 4 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

1. (a) Determine the eigenvalues of the corresponding eigenvectors of the matrix

$A = \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$. (10 marks)

(b) A linear time - invariant system is characterized by the vector - matrix differential equation $\frac{d\underset{\sim}{x}}{dt} = A\underset{\sim}{x}$,

Where $A = \begin{bmatrix} 0 & 1 \\ 3 & -2 \end{bmatrix}$ and $\underset{\sim}{x}(t)$ is the system state vector. Determine the state transition matrix, $\Phi(t)$ of the system. (10 marks)

2. (a) (i) Given that x_n is an approximation to the root of the equation $x^4 - x^2 - 1 = 0$, use the Newton - Raphson method to show that a better approximation is given by $x_{n+1} = \frac{3x_n^4 - x_n + 1}{4x_n^3 - 2x_n}$

(ii) By taking $x_0 = 1.3$, determine the root, correct to four decimal places. (9 marks)

(b) Table 1 represents a polynomial $f(x)$.

Table 1

x	-1	0	1	2	3	4
f(x)	-1	-3	-1	17	87	269

Use the Newton - Gregory interpolation formula to determine:

(i) $f(0.2)$;

(ii) $f(4.6)$. (11 marks)

3. (a) Evaluate the integrals:

(i) $\int_0^2 \int_0^{\sqrt{2x-x^2}} \frac{x}{x^2+y^2} \, dy dx$

(ii) $\int_0^1 \int_0^{\sqrt{1-y^2}} \int_0^{\sqrt{1-x^2-y^2}} z \, dz \, dx dy$ (11 marks)

(b) Use a double integral to determine the area of the region bounded by the curve $x = y^2 - y$ and the line $y = x$. (9 marks)

4. (a) Evaluate the line integral $\int_c x^2 dx + xydy$, where c is the arc of the circle $x^2 + y^2 = 1$ from (0, 1) to (1, 0). (4 marks)

(b) Show that the line integral $\int_{(0,0)}^{(1,\frac{\pi}{2})} (x - \cos y)\, dx + (x \sin y + 2y)\, dy$ is independent of path, and determine its value using a potential function. (7 marks)

(c) Use Green's theorem in the plane to evaluate the line integral

$\oint_c (x^2 - y^2)\, dx + x^2 dy$, where c is the boundary of the triangle with vertices $(0,0)$, $(1,0)$ and $(\frac{1}{2},1)$. (9 marks)

5. (a) Sketch the odd extension of the function $f(t) = 1 - t^2,\ 0 < t < 1$, on the interval $-2 < t < 2$, and determine its half-range Fourier sine series. (7 marks)

(b) Given the function $f(t) = t^2,\ 0 \le t \le 2\pi$,

(i) Sketch the graph of $f(t)$ in the interval $-2\pi < t < 2\pi$.

(ii) Determine the Fourier series representation of $f(t)$. (13 marks)

6. (a) Find the surface area of the part of the cone $z^2 = 16(x^2 + y^2)$ that lies between the planes $z = 0$ and $z = 8$. (5 marks)

(b) Use the Divergence theorem to evaluate the surface integral $\iint_s \underset{\sim}{F}.d\underset{\sim}{s}$ for the vector field $\underset{\sim}{F} = (x - z)\underset{\sim}{i} + (y - x)\underset{\sim}{j} + (z - y)\underset{\sim}{k}$, given that s is the sphere $x^2 + y^2 + z^2 = 9$. (5 marks)

(c) Use Stokes' theorem to evaluate the line integral $\oint_c \underset{\sim}{F}.d\underset{\sim}{r}$ for the vector field $\underset{\sim}{F} = z\underset{\sim}{i} + x\underset{\sim}{j} + y\underset{\sim}{k}$ where c is the boundary of the plane $x + y + z = 2$ in the first octant. (10 marks)

7. (a) Given the function $u = e^{3x} \cos 3y$,

(i) show that u is a harmonic function

(ii) determine a conjugate harmonic function $V(x,y)$ such that $f(z) = u + jv$ is analytic. (10 marks)

(b) The circle $|z| = 1$ is mapped onto the w-plane by the transformation $w = \frac{1}{z + 2j}$. Determine the centre and radius of the image circle. (10 marks)

8. (a) Show that one root of the equation $x^4 + x^2 - 10 = 0$ lies between $x = 1$ and $x = 2$ and use the Newton - Raphson method to determine the root, correct to four decimal places. (8 marks)

(b) The emf produced by a half - wave rectifier is given by

$$e(t) = \begin{cases} 10\sin t & , \quad 0 < t < \pi \\ 0 & , \quad \pi < t < 2\pi \\ e(t + 2\pi) & \end{cases}$$

Determine the Fourier series representation of $e(t)$. (12 marks)

THIS IS THE LAST PRINTED PAGE.