Name:_____ Index No._____

2201/301 2204/301
2203/301 2206/301
MATHEMATICS
Oct./Nov. 2015
Time: 3 hours

Candidate's Signature:_____

Date:_____

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN ELECTRONIC ENGINEERING
DIPLOMA IN TELECOMMUNICATION ENGINEERING
DIPLOMA IN ELECTRICAL ENGINEERING (POWER)
DIPLOMA IN INSTRUMENTATION AND CONTROL ENGINEERING

MATHEMATICS

3 hours

INSTRUCTIONS TO CANDIDATES

Write your name and index number in the spaces provided above.
Sign and write the date of the examination in the spaces provided above.
You should have the following for this examination:
Mathematical tables/ non programmable scientific calculator;
Geometrical drawing instruments.
An abridged table of Laplace Transforms and the Standard Normal Tables are attached.
*This paper consists of **EIGHT** questions.*
*Answer any **FIVE** of the **EIGHT** questions in the spaces provided in this question paper.*
Candidates should answer the questions in English.

For Examiner's Use Only

Question	1	2	3	4	5	6	7	8	TOTAL SCORE
Candidate's Score									

This paper consists of 20 printed pages

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

Turn over

1. (a) Find the eigen values and corresponding eigen vectors of the matrix $A = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}$.

(10 marks)

(b) A linear time - invarient system is characterized by the vector - differential equation,

$\frac{dx}{dt} = Ax$, where $A = \begin{pmatrix} 4 & 0 \\ 1 & -3 \end{pmatrix}$. Find the state transition matrix $\Phi(t)$ of the system.

(10 marks)

2. (a) Find the inverse Laplace transform of f(s), where:

$$f(s) = \frac{5s + 3}{(s - 1)(s^2 + 2s + 5)}.$$

(8 marks)

(b) Use Laplace transforms to find the current $i_2(t)$ in the network of figure 1 assuming the capacitor is uncharged at t = 0.

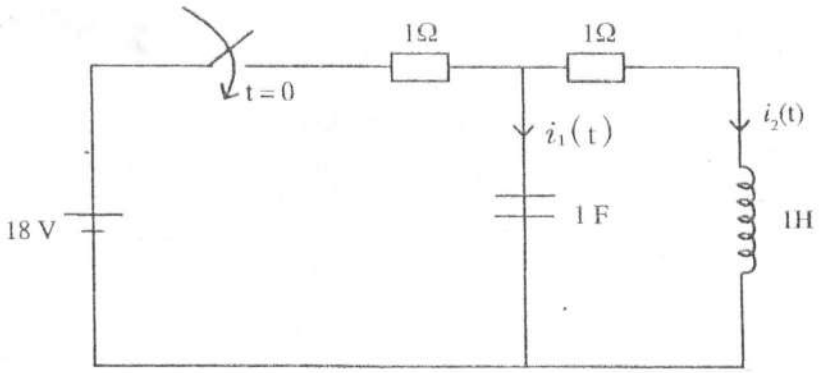


Figure 1

(12 marks)

3. (a) Solve the differential equation $\frac{dx}{dt} + 3x = e^{-2t}$, given that $x = 4$ when t = 0.

(8 marks)

(b) Use the method of undetermined coefficients to solve the differential equation

$\frac{d^2x}{dt^2} + \frac{dx}{dt} - 6x = e^{2t}$, given that $x = 0$, $\frac{dx}{dt} = 1$ when t = 0. (12 marks)

4. (a) Find the half-range Fourier sine series to represent the function $f(x) = 3x$ in the range $0 \le x \le \pi$. (8 marks)

(b) A function f(x) is defined by $f(x) = \begin{cases} 1 + \frac{2x}{\pi}, & -\pi \le x \le 0 \\ 1 - \frac{2x}{\pi}, & 0 \le x \le \pi \end{cases}$:

(i) sketch the graph of $f(x)$ for $-\pi \le x \le \pi$;

(ii) obtain the Fourier series for the function $f(x)$ and deduce that:

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \text{---}$$ (12 marks)

5. (a) Table 1 represents values of x and the corresponding values of $f(x)$:

Table 1

x	0	1	2	3	4
$f(x)$	3	6	11	18	27

(i) use Newton-Gregory interpolation formula to determine $f(x)$;

(ii) hence find f(2.8).

(10 marks)

(b) (i) Using the Newton - Raphson method, show that if X_n is an approximation to $\sqrt[4]{a}$, then a better approximation is given by $X_{n+1} = \frac{1}{4}\left(3X_n + \frac{a}{X_n^3}\right)$.

(ii) Hence evaluate $\sqrt[4]{17}$, correct to six decimal places.

(10 marks)

6. (a) (i) Determine the constants a and b in order that:
$f(z) = (x^2 + ay^2 - 2xy) + j(bx^2 - y^2 + 2xy)$ is analytic.

(ii) Express f(z) and $f'(z)$ in terms of the complex variable $z = x + jy$.

(9 marks)

(b) Find the image in the ω - plane of the circle $|z| = 2$ under the transformation equation:

$$\omega = \frac{z - j}{z + j}.$$ (11 marks)

7. (a) Find the area enclosed by the curve $y = 2 - x$ and $y^2 = 2(2 - x)$ using a double integral. (10 marks)

(b) Verify Green's theorem in the plane for $\oint_c (3x^2 - 8y^2)dx + (4y - 6xy)dy$ where c is the boundary of the region defined by; $x = 0, y = 0, x + y = 1$. (10 marks)

8. (a) A batch of 1800 electric bulbs have a mean life span of 12 hours and a standard deviation of 3 hours. Assuming that the lifespan is normally distributed, determine the number of bulbs with a lifespan of:

(i) more than 15 hours;
(ii) between 10 hours and 14 hours.

(6 marks)

(b) Records from an electronic firm show that the proportion of time t days that a given type of machine is out of service for repairs is given by:

$$f(t) = \begin{cases} ke^{-0.45t}, & t \geq 0 \\ 0, & \text{elsewhere} \end{cases}$$ where k is a constant.

Determine the:

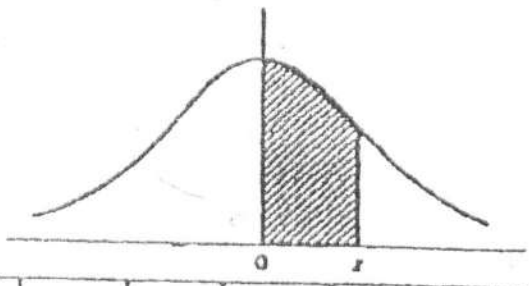
(i) value of the constant k;
(ii) mean and variance of t;
(iii) $p(t \geq 2)$.

(14 marks)

An abridged table of
Laplace Transforms

f(t)	L T F (s)
k (constant)	k/s
t^n (n + ve integer)	$n!/s^{n+1}$
e^{at}	$\frac{1}{s-a}$
cos at	$\frac{s}{s^2+a^2}$
sin at	$\frac{a}{s^2+a^2}$
cosh at	$\frac{s}{s^2-a^2}$
sinh at	$\frac{a}{s^2-a^2}$
te^{-at}	$\frac{1}{(s+a)^2}$
t cos at	$\frac{s^2-a^2}{(s^2+a^2)^2}$
t sin at	$\frac{2as}{(s^2+a^2)^2}$
$f'(t)$	$SF(s) - F(0)$
$f''(t)$	$S^2F(s) - SF(0) - F_1(0)$

Partial areas under the standardised normal curve



$z = \frac{x-\bar{x}}{\sigma}$	0	1	2	3	4	5	6	7	8	9
0.0	0.0000	0.0040	0.0080	0.0120	0.0159	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0678	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1388	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1891	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2086	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2760	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3215	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3451	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3819	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4430	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4762	0.4767
2.0	0.4772	0.4778	0.4783	0.4785	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4882	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4980	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000

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