1. (a) Given $f(z) = z^2 + 3z + 2$,

   (i) express f(z) in the form u + jv.
   (ii) Show that u and v satisfy the Cauchy - Riemann equations. (5 marks)

   (b) Given $u = \text{Cos}hx \cos y$,
   (i) Show that u is a harmonic function.
   (ii) determine a harmonic conjugate function v such that f(z) = u + jv is analytic. (8 marks)

   (c) Find the image of the circle $|z| = 2$ in the w -plane under the transformation

   $$w = \frac{1}{z+j}$$ (7 marks)

2. (a) Determine the eigenvalues and the corresponding eigenvectors of the matrix

   $$A = \begin{bmatrix} 3 & 2 \\ 4 & 5 \end{bmatrix}$$ (10 marks)

   (b) A system is characterized by the vector-matrix differential equation $\frac{dx}{dt} = A\underset{\sim}{x}$,
   where $A = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix}$

   (i) determine the system state transition matrix, $\phi(t)$;
   (ii) show that $\phi(o) = I$, where I is a 2 x 2 unit matrix. (10 marks)

3. (a) Find the half-range Fourier cosine series of the function $f(t) = \pi^2 - t^2, 0<t<\pi$. (8 marks)

   (b) The charge q(t) on the plates of a capacitor is as shown in figure 1.

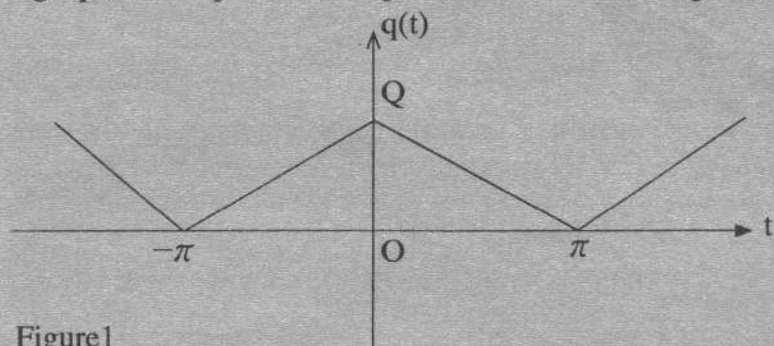

Figure1

   Determine the analytical description of q(t) and find its Fourier series representation. (12 marks)

4. (a) Sketch the domain of integration, and evaluate the integral

   $$\int_0^2 \int_0^{\sqrt{2x-x^2}} \frac{1}{\sqrt{x^2+y^2}}\, dy\, dx$$ (8 marks)

   (b) A region R of the xy - plane is bounded by the curve $y = x^2$ and the lines $y = 0$ and $x = 1$. Show that

   $$\iint_R e^{x^3}\, dx\, dy = \tfrac{1}{3}(e-1)$$ (5 marks)

(c) Use a double integral to determine the area of the region enclosed by the curve $y = -\sqrt{4-x}$ and the straight line $y = \frac{1}{2}x - 2$. (7 marks)

5. (a) (i) Given that x_n is an approximation to the root of the equation $x^3 + 2x - 1 = 0$, Use the Newton - Raphson method that a better approximation is given by

$$x_{n+1} = \frac{2x_n^3 + 1}{3x_n^2 + 2}.$$

(ii) Taking $x_0 = 0.3$, determine the root, correct to four decimal places. (9 marks)

(b) The table below represents a polynomial function f(x):

x	−1	0	1	2	3	4	5
f(x)	−6	−3	0	9	30	69	132

Use the Newton-Gregory interpolation to determine f(x), and find:

(i) f(−1.3);
(ii) f(3.7). (11 marks)

6. (a) Show that the line integral

$\int_{(0,0)}^{(1,\frac{\pi}{2})} 2x\sin y\,dx + (x^2\cos y - 3y^2)\,dy$ is path independent, and use a potential function to determine its value. (8 marks)

(b) Verify Green's theorem in the plane for the line integral $\oint x^2 y\,dx + (x-y)\,dy$, where c is the boundary of the triangle with vertices (0,0), (1,1) and (0,1), with counter clockwise orientation. (12 marks)

7. (a) Determine the area of the paraboloid $z = 2(x^2 + y^2)$ cut off by the cone $Z = \sqrt{x^2 + y^2}$. (10 marks)

(b) Evaluate the surface integral $\iint_s yz\,ds$, where s is the part of the plane $z = y + 3$ that lies inside the cylinder $x^2 + y^2 = 1$. (10 marks)

8. (a) Use the divergence theorem to evaluate $\iint_S \underset{\sim}{F}.d\underset{\sim}{s}$, for the vector field $\underset{\sim}{F} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$ given that S is the sphere $x^2 + y^2 + z^2 = 9$. (10 marks)

(b) Use stokes' theorem to evaluate the line integral $\oint_c \underset{\sim}{F}.d\underset{\sim}{r}$, where the vector field $\underset{\sim}{F} = 3y\underset{\sim}{i} + 4z\underset{\sim}{j} - 6x\underset{\sim}{k}$, and C is the boundary of the paraboloid $z = 9 - x^2 - y^2$ that lies above the xy - plane. (10 marks)

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