2501/103 2508/103
2502/103 2509/103
2503/103
ENGINEERING MATHEMATICS I
June/July 2023
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN MECHANICAL ENGINEERING
(PRODUCTION OPTION)
(PLANT OPTION)
DIPLOMA IN AUTOMOTIVE ENGINEERING
DIPLOMA IN WELDING AND FABRICATION
DIPLOMA IN CONSTRUCTION PLANT ENGINEERING

MODULE I

ENGINEERING MATHEMATICS I

3 hours

INSTRUCTIONS TO THE CANDIDATES

You should have the following for this examination:

Answer booklet;
Mathematical tables/Non-programmable scientific calculator;
Drawing instrument.

*This paper consists of **EIGHT** questions.*
*Answer any **FIVE** questions in the answer booklet provided.*
All questions carry equal marks.
Maximum marks for each part of a question are as shown.
Candidates should answer the questions in English.

This paper consists of 4 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

1. (a) Convert the recurring decimal number $0.\dot{4}0\dot{5}$ into a fraction in its lowest form.

(5 marks)

(b) Determine the sum of the series

$21 + 25 + 29 + ... + 101$

(6 marks)

(c) The average of the first and fourth terms of a geometrical progression is 140. Given that the first term of the series is 64, determine the:

(i) common ratio;
(ii) 8^{th} term;
(iii) sum of infinity.

(9 marks)

2. (a) Given that $Cos\, A = \frac{1}{\sqrt{3}}$, where A is acute, determine the other 5 trigonometric ratios of θ. (6 marks)

(b) Prove the trigonometric identity

$$\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = 2\,\mathrm{cosec}\,\theta$$

(4 marks)

(c) (i) Express $9\cos x - 6\sin x$ in the form $R\cos(x+\alpha)$ where α is an acute angle

(ii) Hence solve the equation $9\cos x - 6\sin x = 5$, where $0° \le x \le 360°$

(10 marks)

3. (a) Given the complex numbers $Z_1 = 3+5j$, $Z_2 = 5+4j$ and $Z_3 = 2-5j$.

determine $Z = Z_3 + \frac{Z_1 + Z_2}{Z_1 - Z_2}$ in the form $a + bj$. (6 marks)

(b) (i) Use De Moivre's theorem to expand $\cos 4\theta$ and $\sin 4\theta$.

(ii) Hence express $\tan 4\theta$ in terms of $\tan\theta$.

(7 marks)

(c) Solve the equation $z^4 + 5 - 12j = 0$ (7 marks)

4. (a) A committee of 7 members is to be formed from 10 ladies and 11 men. Determine the number of ways it can be done to include at least 4 ladies. (5 marks)

(b) (i) Use the binomial theorem to expand $\sqrt{\frac{9+18x}{9-18x}}$ up to the third term.

(ii) By setting $x=\frac{1}{10}$, determine the value of $\sqrt{6}$.

(12 marks)

(c) Determine the constant term in the binomial expansion of

$$\left(3x^2+\frac{1}{5x^3}\right)^5$$

(3 marks)

5. (a) Solve the equation

$$4^x+2^{2x+1}=24$$

(6 marks)

(b) Use elimination method to solve the equations:

$$2x+3y+4z=20$$
$$3x-4y+2z=1$$
$$7x+4y+z=18$$

(9 marks)

(c) Solve the equation

$$\log_2(3x+6)-\log_2(5x-4)=3$$

(5 marks)

6. (a) Determine the logarithmic form of $\tanh^{-1}x$. (7 marks)

(b) Prove the hyperbolic identity

$$\frac{\cosh 2x+\sinh 2x}{\cosh^2 x-\sinh^2 x}=\frac{1+\tanh x}{1-\tanh x}$$

(5 marks)

(c) Solve the equation

$$6\cosh x-8\sinh x=3$$

(8 marks)

7. (a) Prove that:

$$\cosh^{-1}\frac{4}{3}+\sinh^{-1}\frac{1}{4}=\sinh^{-1}(1.242392).$$ (7 marks)

(b) Show that the polar form of the cartesian equation $y^2+6x-9=0$ is given by

$$r=\frac{3}{1+\cos\theta}$$

(6 marks)

(c) Prove that $\tan^{-1}2-\tan^{-1}\frac{1}{3}=\frac{\pi}{4}$.

(7 marks)

8. (a) Evaluate the fraction

$$\frac{\frac{2}{7}\times\frac{1}{6}\div\frac{5}{9}+\frac{3}{4}\text{ of }\frac{1}{2}}{\left(\frac{1}{5}+\frac{2}{3}\right)\times\frac{1}{7}\div\frac{2}{3}}$$

(7 marks)

(b) If $(2x+1)^2+(x-1)(x-3)=ax^2+b$

determine the values of a and b. (4 marks)

(c) The roots of the equation $2x^2-3x+5=0$ are α and β,

determine without solving the equation.

(i) $\alpha^2+\beta^2$

(ii) $(\alpha\beta)^2$

(iii) the equation whose roots are $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$.

(9 marks)

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