2501/203 2508/203
2502/203 2509/203
2503/203
ENGINEERING MATHEMATICS II
June/July 2018
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN MECHANICAL ENGINEERING (PRODUCTION OPTION)
DIPLOMA IN MECHANICAL ENGINEERING (INDUSTRIAL PLANT OPTION)
DIPLOMA IN AUTOMOTIVE ENGINEERING
DIPLOMA IN MECHANICAL ENGINEERING (WELDING AND FABRICATION OPTION)
DIPLOMA IN CONSTRUCTION PLANT ENGINEERING

MODULE II

ENGINEERING MATHEMATICS II

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:
Mathematical tables/Non-programmable scientific calculator;
An answer booklet.
Answer ***FIVE*** *of the following* ***EIGHT*** *questions.*
All questions carry equal marks.
Maximum marks for each part of a question are as shown.
Candidates should answer the questions in English.

This paper consists of 6 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

Turn over

1. (a) Differentiate $f(x)=\frac{1}{2x+3}$ from first principles.

(6 marks)

(b) Find $\frac{dy}{dx}$, given that:

(i) $y=\cos\left(\frac{x^2-2}{x+3}\right)$;

(ii) $x^3y+xy^2-4x^3+6y^2=0$.

(7 marks)

(c) The distance S metres travelled by a body in t seconds is given by

$S=t^3-11t^2+24t-7$

Find the:

(i) values of t when the body is at rest;

(ii) value of t when the acceleration is 2 m/s^2.

(7 marks)

2. (a) Evaluate the integrals:

(i) $\int_0^1 x^2e^{3x}\,dx$

(ii) $\int \frac{x^2+11}{(x-3)(x^2+1)}\,dx$

(10 marks)

(b) Find the position of the centroid of the figure bounded by the curve $y=x^2+4x+2$, the x - axis, and the ordinates at x = 0 and x = 2.

(10 marks)

3. (a) Given that $V=x^2\sin\left(\frac{y}{x}\right)$, show that $x\frac{\partial v}{\partial x}+y\frac{\partial v}{\partial y}=2v$. (5 marks)

(b) Given that $Z=e^{3h}\ln(5h+6p)$. Use partial differentiation to determine the approximate change in Z, if h increases from 0.3 cm to 0.35 cm and p decreases from 0.2 cm to 0.18 cm.

(6 marks)

(c) Locate the stationary points of the function $Z=2x^3-6x^2-8y^2+2$ and determine their nature.

(9 marks)

4. (a) (i) Obtain the first four non-zero terms in the Maclaurin's series expansion of $f(x) = \ln(3x+1)$.

(ii) Hence, evaluate $\int_0^{2} \frac{\ln(3x+1)}{x}\,dx$, correct to four decimal places.

(12 marks)

(b) (i) Use Taylor's theorem to expand $\cos\left(\frac{\pi}{6} + h\right)$ in ascending powers of h as far as the term in h^3.

(ii) Hence, evaluate cos 30.9°, correct to three decimal places.

(8 marks)

5. (a) The sum of three consecutive numbers of an arithmetic progression is 9 and their product is 20.25. Determine the three numbers. (7 marks)

$a-d,\ a,\ a+d = 9$
$(a-d)(a)(a+d) = 20.25$

(b) The ratio of the fourth to the second terms of a geometric progression with a positive common ratio is $\frac{1}{4}$. If the first term exceeds the third term by $7\frac{1}{2}$, determine the:

(i) seventh term;

(ii) sum of the first nine terms.

(8 marks)

(c) A woman deposited Ksh.4,000 in a savings scheme in the first month. Thereafter she increased her deposits by Ksh.400 per month for fifty months. Determine the:

(i) last amount deposited;

$a=4000$
$d=400$
$n=50$

$a+(n-1)d = 4000+(50-1)400$
$= 23,600$

(ii) total amount saved in the fifty months.

$S_n = \frac{n}{2}(2a+(n-1)d)$
$= \frac{50}{2}(2\times4000+(50-1)400)$
$= 690,000$

(5 marks)

6. (a) Table 1 shows marks scored by 170 students in a mechanical science test.

Table 1

Marks x	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70
Frequency f	10	20	x	40	y	25	15

Given that the median mark is 35, determine the:

(i) values of x and y;

(ii) mode.

$x = ?$
$y = ?$

(10 marks)

$4+\left(\frac{1-r^n}{1-r}\right)$

(b) Table 2 shows the masses in kilograms of 40 steel rods used in a mechanical workshop.

Table 2

6 – 6.4 → 3 7.5 – 7.9 → 11 9.0 – 9.4 → 2
6.5 – 6.9 → 5 8.0 – 8.4 → 8
7.0 – 7.4 → 7 8.5 – 8.9 → 4

6.6	8.7	7.9	7.4	8.4	7.2	8.1	7.8	6.8	7.4
8.0	7.1	9.1	6.2	7.7	8.6	8.7	7.2	8.0	7.7
7.6	8.3	7.5	7.1	8.3	6.7	9.4	6.4	8.2	7.8
7.7	6.7	7.6	8.2	7.8	8.8	6.6	7.9	7.4	6.4

(i) Group the data into a frequency distribution using classes of 6.0 - 6.4, 6.5 - 6.9, ...

(ii) Hence, determine the: mean $\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{306}{40} = 7.65$

(I) mean;

(II) standard deviation.

(10 marks)

7. (a) The marks of 250 candidates in a test are normally distributed with a mean of 40 marks and a standard deviation of 10 marks. If 0.4% of the candidates scored x marks and above, determine the:

$\sigma = 10$, $\bar{x} = 40$, $z = \frac{x - \bar{x}}{\sigma}$, $x = \frac{40 - 40}{10} = 0$

(i) value of x;

(ii) number of candidates who scored between 34 marks and 51 marks.

$z_1 = \frac{34-40}{10} = -0.6 \rightarrow 0.2258$

$z_1 = \frac{51-40}{10} = 1.1 \rightarrow 0.3643$

(10 marks)

(b) The diameter of a bolt is assumed to be a continuous random variable with a probability density function.

$$f(x)=\begin{cases} c\,(3x^2 - x^3) & 0 \le x \le 1 \\ 0 & \text{elsewhere} \end{cases}$$

$c\int(3x^2 - x^3)$

$c\left[x^3 - \frac{x^4}{4}\right]_0^1 = 1$

$c\left(1^3 - \frac{1^4}{4}\right) = 1$

$\frac{3}{4}c = 1$

$c = \frac{4}{3} = 1.33$

Determine the:

(i) value of constant C;

(ii) mean; $= \int xf\,x\,dx = C\int x\,dx$

(iii) $P\left(\frac{1}{2} < x \le 1\right)$.

(10 marks)

8. (a) Four coplanar forces $20\angle 120^\circ N$, $40\angle -80^\circ N$, $65\angle 240^\circ N$ and $37\angle 310^\circ N$ act at a point 0. Determine the:

Horizontal forces = 20cos120 + 40cos(-80) + 65cos240 + 37

(i) magnitude;

Vertical forces = 20sin120 − 40sin(-80) + 65sin240 + 37

magnitude = $\sqrt{H^2 + V^2}$

(ii) direction of the resultant force. $\tan\theta = \frac{H}{V}$

(8 marks)

(b) Calculate the total surface area of a hemisphere of a diameter 4.5 cm. $4\pi r^2$ (3 marks)

(c) Use the Simpson's rule with eight intervals to evaluate:

$\int_{0.2}^{1.8} \sqrt{2(1+x^3)}\,dx$ correct to four decimal places.

(9 marks)