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ENGINEERING MATHEMATICS II

June/July 2023

Time: 3 hours



THE KENYA NATIONAL EXAMINATIONS COUNCIL

**DIPLOMA IN MECHANICAL ENGINEERING
(PRODUCTION OPTION)
(PLANT OPTION)**

**DIPLOMA IN AUTOMOTIVE ENGINEERING
DIPLOMA IN WELDING AND FABRICATION
DIPLOMA IN CONSTRUCTION PLANT ENGINEERING**

MODULE II

ENGINEERING MATHEMATICS II

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:

Answer booklet;

Mathematical tables/ Non-programmable scientific calculator.

Answer FIVE of the following EIGHT questions.

All questions carry equal marks.

Maximum marks for each part of a question are as shown.

Candidates should answer the questions in English.

This paper consists of 7 printed pages.

**Candidates should check the question paper to ascertain that
all the pages are printed as indicated and that no questions are missing.**

1. (a) In an arithmetic progression, the sum of the first 9 terms is 3, and the 18th term is three times the 9th term. Determine the:
- (i) common difference;
 - (ii) first term;
 - (iii) sum of the first 20 terms. (10 marks)

- (b) In a geometric progression the fourth term exceeds the fifth term by 4 and the sum of these two terms is 20. Determine the:
- (i) common ratio;
 - (ii) first term;
 - (iii) sum of the first seven terms. (10 marks)

2. (a) From first principles, determine the first derivative of the function.

$$f(t) = e^{2t} \quad (6 \text{ marks})$$

- (b) Investigate the stationary points of $y = \frac{x^3}{3} + \frac{5}{2}x^2 + 6x + 8$ and determine their nature; hence sketch the curve. (14 marks)

3. (a) Use Taylor's theorem to expand $\cos\left(\frac{\pi}{3} + h\right)$ in ascending powers of h up to the term in h^3 . (6 marks)

- (b) (i) Using Maclaurin's theorem; obtain the power series of $\sin x$ up to the term in x^3 .

- (ii) Expand $\frac{1}{\sqrt{1-x^2}}$ using binomial theorem up to the term in x^3 .

- (iii) Hence using (b)(i) and (b)(ii) evaluate

$$\int_0^{\frac{\pi}{4}} \frac{\sin x}{\sqrt{1-x}} dx$$

Correct to three decimal places. (14 marks)

4. (a) Evaluate the integrals:

(i) $\int \frac{dx}{4x^2 + 8x + 7}$

(ii) $\int_3^6 \frac{4x}{(x-1)(x+1)^2} dx$ (12 marks)

- (b) Sketch the region enclosed by the curves $y = \sqrt{x}$ and $y = \frac{x^2}{8}$. Hence determine the area of the enclosed region. (8 marks)

5. (a) Table 1 shows the number of overtime hours claimed in a certain year by production workers in a tyre factory.

Table 1

55	25	57	34	84	18	52
18	20	68	48	56	67	45
64	60	35	37	69	61	71
43	32	63	53	65	24	80
38	30	78	28	75	44	66
23	29	40	44	33	36	31
36	48	50	46	42	48	57
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- (i) construct a frequency distribution table starting from 15 - 25 ... (3 marks)
- (ii) Using the result of (a)(i) draw a histogram, hence determine the modal value. (5 marks)
- (b) Table 2 shows the scores distribution of the candidates interviewed to join a mechanical engineering class.

Table 2

Scores	No. of candidates
10 - 20	60
20 - 30	80
30 - 40	90
40 - 50	110
50 - 60	60
60 - 70	50
70 - 80	40

Determine the:

- (i) mean number of candidates;
- (ii) median;
- (iii) mode;
- (iv) standard deviation. (12 marks)

6. (a) A manufacturer knows that the components he makes contain an averages 0.8% of defectives. He packs them in packets of 6. Determine the probability that a packet picked at random will contain 4 or more faulty components. (4 marks)

- (b) A random variable x has a probability density function defined by:

$$f(x) = \begin{cases} kx & , 0 \leq x \leq 1 \\ k(2-x) & , 1 \leq x \leq 2 \\ 0 & , \text{elsewhere} \end{cases}$$

Determine the:

- (i) value of the constant k ;
(ii) mean;
(iii) variance;
(iv) $\Pr\left(\frac{1}{4} < x < \frac{3}{2}\right)$. (16 marks)

7. (a) Show that the three vectors:

$$\underline{a} = 2\underline{i} - \underline{j} + \underline{k}$$

$$\underline{b} = \underline{i} - 3\underline{j} - 5\underline{k}$$

$$\underline{c} = 3\underline{i} - 4\underline{j} - 4\underline{k}$$

form the sides of a right angled triangle. (10 marks)

- (b) A particle experience forces of magnitude 2, $4\sqrt{2}$, 6 and 8 inclined at angles of 30° , 45° , 60° and 120° respectively to a given reference direction ox . By using resolution of vectors, find the:

- (i) magnitude;
(ii) direction of the resultant force. (10 marks)

8. (a) Figure 1 shows a solid made up of two parts : a cylindrical bottom and a conical top. The base has a radius R and height H , with a top truncated cone portion whose topmost radius is r and a vertical height of h form the top of the cylinder. It also has a semi-spherical base ABC as shown.

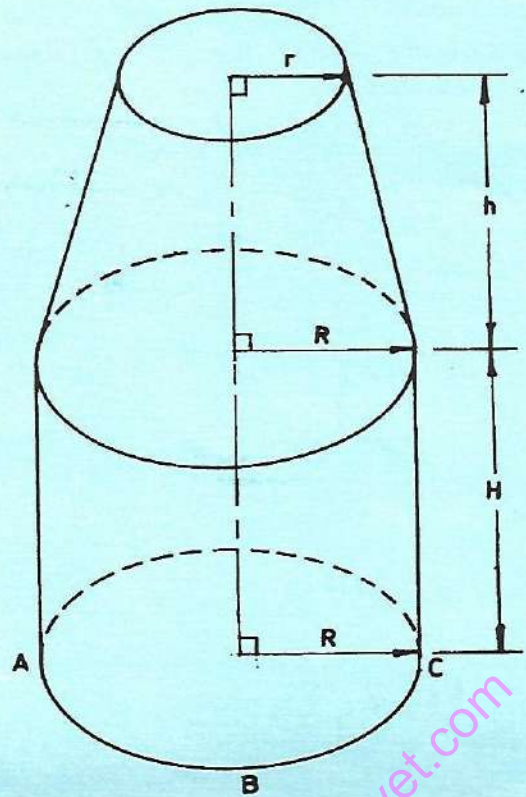


Fig.1

Taking $r = 4$ cm, $R = 7$ cm, $h = 3$ cm and $H = 10$ cm. Determine the:

- (i) total surface area;
- (ii) volume of the solid in litres.

(12 marks)

(b) Figure 2 shows a regular solid with $\angle CAD = 34^\circ$, $\angle CBD = 28.5^\circ$ and $\triangle ADB$, $\triangle CDA$ and $\triangle CDB$ are right angled at D . Given that $AB = 22.5$ cm; determine:

- (i) height, h
- (ii) volume of the solid $ABCD$ correct to four decimal places. (8 marks)

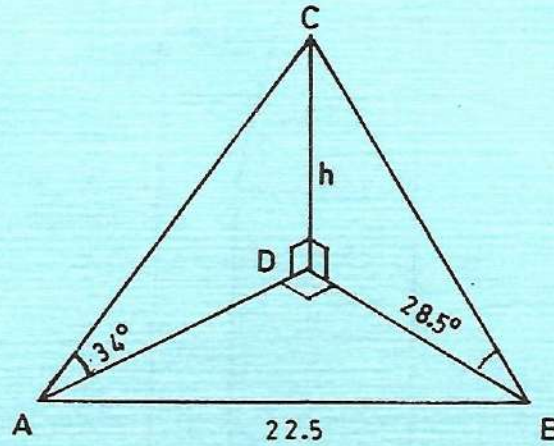


Fig 2